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NONLOCAL IMPROVEMENT OF CONTROLS IN DYNAMICAL SYSTEMS QUADRATIC WITH RESPECT TO STATE

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1. Introduction

We consider the following optimal control problem

$$\Phi(u) = \varphi(x(t_1)) + \int_T F(x(t), u(t), t)dt \to \min_{u\in V}, \tag{1}$$

$$\dot{x} = f(x, u, t), \quad x(t_0) = x^0, \quad u(t) \in U, \quad t \in T = [t_0, t_1], \tag{2}$$

in which $x(t) = (x_1(t), \dots, x_n(t))$ is the vector of state, $u(t) = (u_1(t), \dots, u_m(t))$ the vector of control. Admissible controls are assumed to belong to a set V of piecewise continuous functions with values in a compact set $U \subset R^m$. The functions $f(x, u, t)$, $F(x, u, t)$ are quadratic with respect to x with coefficients continuously depending on u, t on the set $R^n \times U \times T$. The function $\varphi(x)$ is quadratic on R^n.

In this article we suggest methods of construction of nonlocal improvements of control, which are based on some exact (without remainder terms) formulas of increment of functional in the class of problems under consideration. The improvement of controls by exact formulas of functional's increment allows us to omit the procedure of the traditional parametric search of an improving approximation in a local neighborhood of a current control.

The approach to the construction of nonlocal improvements of control generalizes the methods of nonlocal improvement in systems linear by state (see [1]).

2. Formulas of increment of functional

Let $u, v \in V$ be a pair of controls with the corresponding phase trajectories $x(t, u)$, $x(t, v)$ of system (2), $\Delta x(t) = x(t, v) - x(t, u)$ be the phase increment, $\Delta_v\Phi(u) = \Phi(v) - \Phi(u)$ the increment of the functional.

We form the Pontryagin function $H(\psi, x, u, t) = \langle\psi, f(x, u, t)\rangle - F(x, u, t)$ and denote $\Delta_v H(\psi, x, u, t) = H(\psi, x, v, t) - H(\psi, x, u, t)$. We denote by H_x, φ_x, H_{xx}, φ_{xx} the first and second order derivatives of the functions H, φ with respect to x.

Let us consider the vector standard conjugate system

$$\dot{\psi} = -H_x(\psi, x, u, t) \tag{3}$$

and introduce the vector modified conjugate system

$$\dot{p} = -H_x(p, x, u, t) - \frac{1}{2}H_{xx}(p, x, u, t)y. \tag{4}$$

Let $\psi(t, u)$, $t \in T$, be a solution of system (3) for $u = u(t)$, $x = x(t, u)$, $\psi(t_1, u) = -\varphi_x(x(t_1, u))$; $p(t, u, v)$, $t \in T$, a solution of system (4) for $u = u(t)$, $x = x(t, u)$, $y = \Delta x(t)$, $p(t_1, u, v) = -\varphi_x(x(t_1, u)) - \frac{1}{2}\varphi_{xx}(x(t_1, u))\Delta x(t_1)$; $p(t, v, u)$, $t \in T$, a solution of system (4) for $u = v(t)$,