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An Approach for Computing Eigenvalues of Discrete Symplectic Boundary-Value Problems

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Abstract—We propose a new method for calculating eigenvalues of discrete symplectic boundary-value problems. This method is based on the discrete oscillation theory and on a modification of the Abramov double sweep method for discrete self-adjoint boundary-value problems.

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1. Introduction. A. A. Abramov [1–3] has proposed a new method for calculating eigenvalues and eigenfunctions of boundary-value problems for Hamiltonian systems of ordinary differential equations with a nonlinear dependence on the spectral parameter. This method is based on certain versions of the differential sweep method intended for the solution of self-adjoint boundary-value problems.

In this paper we propose a new method for computing eigenvalues of discrete symplectic boundary-value problems

$$y_{i+1} = W_i(\lambda)y_i,\ \ i = 0,\dots,N,\ \ y_i = [x_i\, u_i]^\top \in \mathbb{R}^{2n}, \tag{1}$$

$$x_0 = x_{N+1} = 0 \tag{2}$$

with a linear dependence on the spectral parameter:

$$W_i(\lambda) = \begin{bmatrix} I & 0 \\ -\lambda w_i & I \end{bmatrix} S_i,\ \ S_i^\top J S_i = J,\ \ J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix},\ \ w_i = w_i^\top,\ \ w_i \ge 0, \tag{3}$$

where $\lambda \in \mathbb{R}$, while I and 0 are identical and zero $n \times n$ matrices, respectively.

As s special case of systems (1), (3), we consider Hamiltonian systems of difference equations

$$J^\top \Delta \begin{bmatrix} X_i \\ U_i \end{bmatrix} = \begin{bmatrix} \lambda w_i - \mathcal{C}_i & \mathcal{A}_i^\top \\ \mathcal{A}_i & \mathcal{B}_i \end{bmatrix} \begin{bmatrix} X_{i+1} \\ U_i \end{bmatrix},\ \ \mathcal{C}_i = \mathcal{C}_i^\top,\ \ \mathcal{B}_i = \mathcal{B}_i^\top,\ \ \det(I - \mathcal{A}_i) \ne 0,$$

which are discrete analogs of Hamiltonian systems of differential equations with a linear dependence on λ.

Let us note that according to [4] problem (1), (2) under consideration is self-adjoint, and the set of all λ such that the corresponding solution $y_i(\lambda) = [x_i\, u_i]^\top$ to problem (1) satisfies the condition $\sum\limits_{i=0}^{N-1} x_{i+1}^\top w_i x_{i+1} > 0$ is a finite set of real numbers.

Our algorithm is based on recent results of the discrete oscillation theory for systems (1). This theory is being actively developed in recent years [4–11]. As a special case, this theory includes the

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