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The Numerical Experiment in the Study of Polynomial Quasisolutions of Linear Differential-Difference Equations

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Abstract—In this paper we consider the initial problem with an initial point for a scalar linear inhomogeneous differential-difference equation of neutral type. For polynomial coefficients in the equation we introduce a formal solution, representing a polynomial of a certain degree ("a polynomial quasisolution"); substituting it in the initial equation, one obtains a residual. The work is dedicated to the definition and the analysis (on the base of numerical experiments) of polynomial quasisolutions for the solutions of the initial problem under consideration.

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Key words and phrases: *linear differential-difference equations, method of polynomial quasisolutions, numerical experiment.*

1. INTRODUCTION

The study of a certain phenomenon by mathematical methods in many cases is reduced to the investigation of various differential equations. The class of functional differential equations allows one to take into account the influence of the previous state on the solutions of the investigated phenomenon. In this class, the differential-difference equations (DDE) with a constant deviation of the argument ([1–3]) are mostly investigated.

In this paper, we consider the scalar inhomogeneous DDE of neutral type

$$\frac{dx(t)}{dt} + p(t)\frac{dx(t-1)}{dt} = a(t)x(t-1) + f(t), \quad t \in J = (-\infty, \infty). \tag{1.1}$$

Let at $t = 0$ the initial condition $x(0) = x_0$ be stated. We have to study solutions which are analytic on J such that being substituted into the initial Eq. (1.1) they turn it into the identity.

It is well-known that the derivatives of a solution to the initial problem for Eq. (1.1) have discontinuities at points which are multiple of the delay, when the initial function $\varphi(t)$ is defined on the initial set $t \in E_{t_0} = [t_0 - 1, t_0]$. However, in many applied problems which are described by Eq. (1.1) the observed structure of a solution is sufficiently smooth. Therefore the study of the initial problem with an initial point for Eq. (1.1) represents an actual problem of great applied importance. We are not aware of solvability conditions for this problem in the class of analytic functions, when Eq. (1.1) has variable coefficients.

In the case, when the coefficients of the equation are represented by polynomials, one introduces a formal solution in the form of a polynomial of a certain degree N. Then the term "a polynomial quasisolution" (a PQ-solution) means that its substitution into the initial problem yields the residual $\Delta(t) = O(t^N)$.

This paper is devoted to the definition and the analysis (on the base of numerical experiments) of PQ-solutions of the initial problem with an initial point for the investigated equation.

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