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Three-Webs $W(1, n, 1)$ and Associated Systems of Ordinary Differential Equations

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Abstract—We consider a three-web on a smooth manifold formed by two n-parameter families of curves and a one-parameter family of hypersurfaces. For such webs, we define a family of adapted frames, find the system of structure equations, and study the differential-geometric objects that arise in differential neighborhoods up to the third order. We prove that each system of ordinary differential equations (SODE) uniquely defines a three-web. This allows us to describe properties of a SODE in terms of the corresponding three-web. In particular, we characterize autonomous SODE.

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The study of the classical geometry of three-webs formed by foliations of the same dimensions originated in the 1920s in works by W. Blaschke. The modern theory of webs was developed by M.A. Akivis and his colleagues and disciples since 1960s. The main results of this theory can be found in [1, 2].

The theory of webs formed by foliations of different dimensions was studied by M. A. Akivis [1–4], V. V. Goldberg [1, 2, 5], and A. M. Vasiliev's disciples N. Kh. Azizova, Yu. A. Apresyan, Nguen Zoan Tuan, and G. A. Tolstikhina ([4], Chap. 9).

In this paper, we consider a three-web $W(1, n, 1)$ formed by two families of curves and a family of hypersurfaces. We find the structure equations of a web in question, their first and second differential prolongations, the structure tensors of a web, and the conditions under which an affine connection arises on a web. We show that to a first order SODU $\dot{x} = f(x, t)$ a three-web $W(1, n, 1)$ is associated. The components of the tensors of a web are expressed in terms of the functions defining the SODU. We show that there arises an affine connection naturally associated to a SODU. We call it the canonical affine connection. In terms of the web, we establish the conditions under which a SODU is autonomous.

1. Let M be a smooth $(n + 1)$-dimensional manifold. Consider on M a three-web $W(1, n, 1)$ formed by two families of curves λ_1, λ_3 and a family of hypersurfaces λ_2. Denote by $T_p(M)$ the tangent space to M at p, and by $T_p(\mathcal{F}_\alpha)$, $\alpha = 1, 2, 3$, the tangent spaces at p to the leaves $\mathcal{F}_\alpha$ of $W(1, n, 1)$. Consider at p a manifold of frames e_a, $a, b, \ldots = 1, 2, \ldots, n + 1$, satisfying the requirement that the first n vectors of a frame lie in $T_p(\mathcal{F}_2)$, the vector e_{n+1} lies in $T_p(\mathcal{F}_1)$, and the vector $e_n - e_{n+1}$ lies in $T_p(\mathcal{F}_3)$. Let ω^a be the dual coframe, i.e.,

$$\omega^a(e_b) = \delta^a_b,$$

where δ^a_b is the Kronecker symbol. Then, for any vector ξ of T_p, we have

$$\xi = \omega^a(\xi)\, e_a, \tag{1}$$

which means that the families λ_1 and λ_2 of the web are given by the following Pfaff equations:

$$\lambda_1 : \omega^i = 0; \quad \lambda_2 : \omega^{n+1} = 0 \quad (i, j, \ldots = 1, 2, \ldots, n). \tag{2}$$

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