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# Calculation of Eigenvalues of a Discrete Self-Adjoint Operator Perturbed by a Bounded Operator

**I. I. Kinzina**[1*]

[1] *pr. Lenina 114, Magnitogorsk, 455038 Russia*

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**Abstract**—We generalize the method of regularized traces which calculates eigenvalues of a perturbed discrete operator for the case of an arbitrary multiplicity of eigenvalues of the unperturbed operator. We obtain a system of equations, enabling one to calculate eigenvalues of the perturbed operator with large ordinal numbers. As an example, we calculate eigenvalues of a perturbed Laplace operator in a rectangle.

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## INTRODUCTION

Regularized traces of the order $p \in \mathbf{N}$ obey the formulas

$$\sum_{n=1}^{\infty}[\lambda_n^p - A_p(n)] = B_p, \tag{0.1}$$

where $\lambda_n$ are eigenvalues of the operator $A$; $A_p(n)$ are known numbers which provide the convergence of numerical series; numbers $B_p$ can be calculated explicitly with the help of the operator characteristics. One tried to apply them for an approximate calculation of first eigenvalues of the operator $A$. Actually, formulas (0.1) allow one to write the algebraic system of equations

$$\sum_{n=1}^{m}[\widetilde{\lambda}_n^p - A_p(n)] = B_p, \;\; p = \overline{1, m}, \tag{0.2}$$

connecting approximate values $\{\widetilde{\lambda}_n\}_{n=1}^m$ of first $m$ eigenvalues $\{\lambda_n\}_{n=1}^m$ of the operator $A$.

In several cases nonlinear system (0.2) enables one to calculate approximately first eigenvalues $\{\widetilde{\lambda}_n\}_{n=1}^m$ of the operators with satisfactory accuracy. But this fact does not substantiate this way as a method for the approximate computation of first eigenvalues, because one truncates the residues of convergent numerical series and do not estimate them. In addition, no universal algorithm exists, calculating $A_p(n)$ and $B_p$ for a wide class of operators. One can find them by the known methods either only in the Sturm–Liouville spectral problems, having the data on the asymptote of eigenvalues [1], or using the data on the recursive Green functions in spectral problems for operators with kernel resolvents. In many cases their determination represents a complex mathematical problem [2].

In 1957 I. M. Gelfand and L. A. Dikii [3] proposed a new method for the approximate calculation of eigenvalues of the Sturm–Liouville operator. It implies that in the considered system of regularized traces one retains partial sums up to the $N$th addend. However, this method was not substantiated theoretically; as an example, I. M. Gelfand and L. A. Dikii calculated first three eigenvalues of the Mathieu equation with satisfactory accuracy. Later S. A. Shkarin [4] proved the non-uniqueness of the

*E-mail: kinzina@masu.ru.