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Manifold of Nondegenerate Affinor Fields

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Abstract—We consider the quotient set of the set of nondegenerate affinor fields with respect to the action of the group of nowhere vanishing functions. This set is endowed with a structure of infinite-dimensional Lie group. On this Lie group, we construct an object of linear connection with respect to which all left-invariant vector fields are covariantly constant (the Cartan connection).

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In the present paper we consider the manifold T_0 of nondegenerate tensor fields of type $(1,1)$ of finite differentiability class on a compact manifold M. On the manifold T_0 we have an action of the group F_0 of functions which do not vanish on M. The quotient manifold T_0/F_0 with respect to this action is an infinite-dimensional Lie group. On each Lie group one can take the Cartan connection with respect to which all the left-invariant vector fields are covariant constant.

On each Lie group Cartan connections of three types exist: the left, the right, and the mean connection ([1], pp. 101—105). All the three Cartan connections are not symmetric, they have zero curvature tensor and nonzero torsion. In the present paper we construct the object of left Cartan connection on the manifold of nondegenerate affine fields.

Definition 1 ([2], pp. 229, 230)**.** A set G is called a *Lie group*, if 1) G is an algebraic group, 2) G is a manifold of class C^∞, 3) the operation of multiplication and the operation of taking the inverse have class C^∞.

Definition 2 ([1], pp. 100, 101)**.** Let G be a Lie group, g be the Lie algebra of G. Then a linear connection ∇ is called the *Cartan connection* on G if $\nabla X = 0 \ \ \forall X \in g$.

Definition 3 ([3], P. 52, 5.7.1)**.** A morphism of manifolds $f : L \to M$ of class C^1 is called an *immersion* at a point $u \in L$ if the linear map $Tf_u : T_uL \to T_{f(u)}M$ is injective and its image is a closed vector subspace in $T_{f(u)}M$ admitting a topological complement.

Theorem ([ibid.], P. 72, 6.2.3)**.** *Let G be a Lie group, M be a manifold of class C^∞ and $\rho : (g,x) \in G \times M \to gx \in M$ be a left action of class C^∞. If*

1) *G acts on M freely and properly,*

2) *for any point $x \in M$, $\rho_x : g \in G \to gx \in M$ is an immersion,*

then the quotient set M/G can be endowed by a unique structure of smooth manifold such that $\pi : M \to M/G$ is a submersion and $(M, \pi, M/G, G)$ is a left principal bundle.

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