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The Exact Boundaries of the Stability Domains of Linear Differential Equations with Distributed Delay

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Abstract—For a differential equation with a distributed varying delay, sufficient criteria for the asymptotic and uniform stability of solutions are obtained. The constructed examples demonstrate exactness of the boundary of the obtained stability domain.

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1. INTRODUCTION

For equations with concentrated varying delay in the form

$$\dot{x}(t) + x\big(t - r(t)\big) = 0, \quad t \geq 0,$$

the following stability criterion is well-known (see [1]): If $0 \leq r(t) \leq \omega < 3/2$, then a solution of the equation is asymptotically stable; if $0 \leq r(t) \leq 3/2$, then a solution of the equation is stable in the sense of Lyapunov. This criterion is remarkable in the fact that the constant $3/2$ used in it is unimprovable. This means that its infinitesimal increase enables one to find a delay $r(t)$ which makes a solution unstable. The mentioned criterion is a base for many generalizations [2–5]. the researchers try to preserve the exactness of the boundary of the stability domain even for more general objects.

In this paper we consider a differential equation with *distributed* varying delay and make an attempt to establish for it the stability criteria analogous to those adduced above.

2. PROBLEM DEFINITION

Let $\mathbb{N}$ stand for the set of natural numbers, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}_+ = [0, \infty)$, $\Delta = \{(t,s) \in \mathbb{R}_+^2 : t \geq s\}$. We denote by $L = L(\mathbb{R}_+)$ the space of summable functions, we do by $L_\infty = L_\infty(\mathbb{R}_+)$ the space of functions which are summable and bounded in essence on $\mathbb{R}_+$; the norm is assumed to be natural.

Consider the following equation:

$$\dot{x}(t) + \int_{t-r(t)}^{t} x(s)\,ds = f(t), \quad t \in \mathbb{R}_+ \tag{1}$$

$$(x(\xi) = 0 \text{ for } \xi < 0).$$

Assume that the delay $r : \mathbb{R}_+ \to \mathbb{R}_+$ is measurable and bounded on $\mathbb{R}_+$, and the function $f : \mathbb{R}_+ \to \mathbb{R}$ is locally summable. With $f \equiv 0$ we call Eq. (1) *homogeneous*.

We understand a *solution* of Eq. (1) as a function $x : \mathbb{R}_+ \to \mathbb{R}$ which is absolutely continuous on each finite interval $[0, T]$ and satisfies (1) almost everywhere.

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