Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 1, pp.81–84, 1998 *UDC 519.71:95*

POLYNOMIAL EXPANSIONS OF k-VALUED FUNCTIONS IN OPERATORS OF DIFFERENTIATION AND NORMALIZATION

V.I. Panteleyev

The various representations for k-valued functions (functions of k-valued logic, or k-valued Boolean functions) are of interest in connection with their use in discrete computational devices (see [1]).

As it is known, an arbitrary k-valued function is representable as a polynomial in $\bmod k$ if number k is prime. By a *polynomial form* we shall call the sum in $\bmod k$ of a finite number of terms constructed in certain way.

Recently there were obtained various polynomial representations for the Boolean functions (see [2]). In the present article we consider a generalization of those results to the case of the k-valued Boolean functions for a prime number k.

In what follows the notation $f(x_1, \dots, x_n)$ is understood as a k-valued function $f(x_1, \dots, x_n)$ for a prime k. The used below operations "+", "$\cdot$" are the addition and multiplication in $\bmod k$, respectively. The operation "$-$" is defined as the inversion of the operation "+". The operations $x^{(\alpha)}$ are defined as follows:

$$x^{(\alpha)} = k - 1 - x + \alpha.$$

We shall say that a function $f(x_1, \dots, x_n)$ is *non-degenerate* if

$$\sum_{\beta_1 \dots \beta_n} f(\beta_1, \dots, \beta_n) \neq 0, \quad \beta_i \in \{0, 1, \dots, k-1\},$$

otherwise it is said to be *degenerate*. For $k = 2$, a function is non-degenerate if and only if its vector of values contains an odd number of units. In the general situation, a function $f(x_1, \dots, x_n)$ is non-degenerate if and only if the polynomial which represents this function has the degree equaling $n(k-1)$.

For an n-ary function $f(x_1, \dots, x_n)$ we define by induction the operators of differentiation (**d**), normalization (**p**), and those which are mixed with respect to (**p**) and (**d**):

$$p^{\beta}_{x_j} f(x_1, \dots x_j, \dots, x_n) = f(x_1, \dots x_j^{(\beta)}, \dots, x_n), \quad 0 \le \beta \le k-1;$$

$$d^{0}_{x_j} f(x_1, \dots x_j, \dots, x_n) = f(x_1, \dots x_j, \dots, x_n);$$

$$d^{\beta}_{x_j} f(x_1, \dots x_j, \dots, x_n) = f(x_1, \dots x_j, \dots, x_n) + f(x_1, \dots x_j - \beta, \dots, x_n), \quad 1 \le \beta \le k-1;$$

$$h^{\beta_1, \dots, \beta_m}_{x_1, \dots, x_m} f(x_1, \dots, x_m, \dots x_n) = h^{\beta_1}_{x_1}(h^{\beta_2, \dots, \beta_m}_{x_2, \dots, x_m} f(x_1, \dots, x_m, \dots x_n)), \quad m \le n, \quad h \in \{p, d, t\}.$$

©1998 by Allerton Press, Inc.
Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of $50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.