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CONTINUOUS FUNCTIONALS AND MULTIPLIERS OF POWER SERIES OF A CLASS OF FUNCTIONS ANALYTIC IN A POLYDISK

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Introduction

Let $U^n = \{z = (z_1, \dots, z_n),\ |z_j| < 1,\ j = 1, \dots, n\}$ be the unit polydisk of the n-dimensional complex space $\mathbb{C}^n$, $\mathbb{T}^n = \{z : |z_j| = 1,\ j = 1, \dots, n\}$ its skeleton, $H(U^n)$ a set of all functions holomorphic in U^n. In what follows we shall use the standard notation: $I^n = [0,1]^n$, $(1-R)^\gamma = (1-R_1)^\gamma \cdots (1-R_n)^\gamma$, $-\infty < \gamma < \infty$, $R_j \in (0,1)$, $j = 1, \dots, n$, $sz = (s_1 z_1, \dots, s_n z_n)$, $s \in \mathbb{R}^n$, $z \in \mathbb{C}^n$, $dR = dR_1 \cdots dR_n$, $M_q(f,r) = \Big(\int\limits_{\mathbb{T}^n} |f(r\xi)|^q dm_n(\xi)\Big)^{1/q}$, $0 < q \le \infty$, $dm_{2n}(z)$ is the Lebesgue measure on U^n, $H^s(U^n)$ the Hardy class in the polydisk U^n, $0 < s \le \infty$. For arbitrary $0 < p, q < \infty$, $-1 < \alpha < \infty$, we introduce the spaces

$$F^{p,q}_\alpha(U^n) = \bigg\{ f \in H(U^n) : \|f\|^p_{F^{p,q}_\alpha(U^n)} = \int_{\mathbb{T}^n} \bigg(\int_{I^n} |f(R\xi|^q (1-R)^\alpha dR \bigg)^{p/q} dm_n(\xi) < \infty \bigg\},$$

where $m_n(\xi)$ is the normed Lebesgue measure on $\mathbb{T}^n$.

One can easily show that with $\max(p,q) \le 1$ the space $F^{p,q}_\alpha$ is a complete metric space with the metric $\rho(f,g) = \|f-g\|^{\min\{p,q\}}$, and with $\min\{p,q\} \ge 1$, with respect to the norm $\|f\|_{F^{p,q}_\alpha}$ the space $F^{p,q}_\alpha$ is Banach. Note that $F^{p,p}_\alpha = A^p_\alpha$, where $A^p_\alpha(U^n)$ are known Bergman–Djrbashian classes (see [1]), while $F^{p,2}_\alpha(U) = H^p_\alpha$, $0 < p < \infty$ (see [2]), where H^p_α are the Hardy–Sobolev classes. Some properties of the introduced spaces and their smooth analogs for $1 < p, q < \infty$ were investigated in [3] and [4].

The objective of this article is the study of some properties of the spaces $F^{p,q}_\alpha$ for $0 < p, q \le 1$, $-1 < \alpha < \infty$.

Let $0 < p, q \le 1$, $-1 < \alpha < \infty$. We denote by $S^{p,q}_\alpha$ the set of all functions holomorphic in U^n and such that, for each $\beta > \frac{\alpha+1}{q} + \frac{1}{p}$,

$$\|f\|_{S^{p,q}_\alpha} = \sup_{z \in U^n} \{|D^{\beta+1} g(z)|(1-|z|)^{\beta+2-\frac{(\alpha+1)}{q}-\frac{1}{p}}\} < \infty,$$

where

$$D^\beta g(z) = \sum_{|k|\ge 0} \frac{\Gamma(k+\beta+1)}{\Gamma(\beta+1)\Gamma(k+1)} c_k z^k, \quad \Gamma(k+\beta+1) = \prod_{j=1}^n \Gamma(k_j+\beta+1), \quad g(z) = \sum_{|k|\ge 0} c_k z^k, \quad \beta > -1,$$

is a fractional derivative of the function $g(z)$. One can easily see that, with respect to norm $\|\cdot\|_{S^{p,q}_\alpha}$ the space $S^{p,q}_\alpha$ is Banach.