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ON RESOLVABILITY OF THE TWO-POINT BOUNDARY VALUE PROBLEM FOR A NONLINEAR SINGULAR FUNCTIONAL-DIFFERENTIAL EQUATION

M.Zh. Alves

1. Principal definitions

Consider a boundary value problem for the following quasilinear singular functional-differential equation

$$\pi(t)\ddot{x}(t) = f(t, (\theta x)(t)), \quad x(0) = \alpha_1, \quad x(1) = \alpha_2, \quad t \in [0, 1], \tag{1}$$

where $\pi(t) = t$, or $\pi(t) = 1 - t$, or $\pi(t) = t(1 - t)$, $\theta : \mathbf{C} \to \mathbf{L}_p$, $1 \le p \le \infty$, is a linear isotonic operator (i. e., if $x(t) \ge y(t)$, then $(\theta x)(t) \ge (\theta y)(t)$ almost everywhere on $[0, 1]$). Here and in what follows $\mathbf{C}$ is the space of continuous functions on $[0, 1]$ with the norm

$$\|x\|_{\mathbf{C}} = \max_{t\in[0,1]} |x(t)|,$$

$\mathbf{L}_p$ for $1 \le p < \infty$ is the space of summable in p-th degree on $[0, 1]$ functions with the norm

$$\|x\|_{\mathbf{L}_p} = \Big(\int_0^1 |x(t)|^p dt \Big)^{1/p},$$

and $\mathbf{L}_\infty$ is the space of measurable essentially bounded on $[0, 1]$ functions with the norm

$$\|x\|_{\mathbf{L}_\infty} = \operatorname{vrai\,sup}_{t\in[0,1]} |x(t)|.$$

By using the scheme of "$\mathcal{L}^1\mathcal{L}^2$-quasilinearization" by N.V. Azbelev (see [1], p. 217), problem (1) can be reduced to an equivalent equation of the second kind with an isotonic operator, to which the Schauder fixed point principle is applicable.

We further reproduce the introductory part of the preceding article [2].

We denote by J_π the interval $(0, 1)$ if $\pi(t) = t(1 - t)$, the interval $(0, 1]$ if $\pi(t) = t$, and the interval $[0, 1)$ if $\pi(t) = 1 - t$.

In following [3], we denote by $\mathbf{D}^p_\pi$ $(1 \le p \le \infty)$ the space of functions $x : [0, 1] \to \mathbf{R}^1$, possessing the properties

a) x is continuous on $[0, 1]$,
b) $\dot{x}$ is continuous on J_π,
c) the product $\pi\ddot{x}$ belongs to $\mathbf{L}_p$.

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