ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 8, pp. 33–44. © Allerton Press, Inc., 2007.
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Integral Equations in the Wave Scattering Problem at an Irregular Interface of Domains

E. K. Lipachev*

Kazan State University, ul. Kremlyovskaya 18, Kazan, 420008 Russia

Received May 23, 2006

Abstract—Modeling the scattering of electromagnetic waves at an interface of media with different characteristics, one encounters the conjugation problem. Using the method of boundary integral equations and the theory of generalized potentials, we prove the classical resolvability of this problem. The boundary is assumed to be irregular. This means that the plane is divided into two domains by a curve which coincides with a straight line, except for a finite part, producing the irregularity. We propose algorithms for the approximate solution of the conjugation problem based on the spline methods for the solution of integral equations. We theoretically substantiate the computational scheme, namely, we prove the convergence and estimate the convergence rate.

DOI: 10.3103/S1066369X0708004X

1. PROBLEM DEFINITION

Let the plane $\mathbb{R}^2$ be divided by a curve Γ into the domains S_1 and S_2. Assume that $\Gamma = \{(x, f(x)) : x \in \mathbb{R}\}$, where $f(x) \in C^{1,\alpha}(\mathbb{R})$, $0 < \alpha \leqslant 1$ and $\operatorname{supp} f \subseteq [-d, d]$ for some positive real number d; in addition, $f(\pm d) = 0$. Thus, the curve Γ coincides with a straight line, except for a finite irregular part. We use the term "irregular boundary of a domain" just in this sense (see also [1, 2]). Denoting the irregular part of the curve Γ by Γ^*, we write

$$\Gamma = \Gamma^* \cup \{(x, 0) : x \notin [-d, d]\}, \quad \Gamma^* = \{(x, f(x)) : x \in [-d, d]\},$$
$$S_1 = \{(x, z) : z > f(x), x \in \mathbb{R}\}, \quad S_2 = \{(x, z) : z < f(x), x \in \mathbb{R}\}.$$

Let $\nu = \nu(M)$ stand for the unit vector of the normal to the curve Γ at the point M directed inwards the domain S_1; let $\partial_\nu = \partial_{\nu(M)}$ stand for the regular normal derivative at the point M (e. g., [3], p. 299; [4], p. 140). With the help of the indices "$-$" and "$+$" we distinguish between the limit values of the functions at the boundary Γ which are calculated from the domains S_1 and S_2, respectively.

Consider the following boundary-value problem. *Find a pair of functions $u_1(x, z)$, $u_2(x, z)$ defined in the domains S_1 and S_2, respectively, such that*

$$\begin{aligned} \triangle u_1(x, z) + k_1^2 u_1(x, z) = 0, \quad (x, z) \in S_1, \\ \triangle u_2(x, z) + k_2^2 u_2(x, z) = 0, \quad (x, z) \in S_2; \end{aligned} \tag{1}$$

at the interface of the domains the limit values of the functions $u_j(x, z)$ and $\partial_\nu u_j(x, z)$ $(j = 1, 2)$ satisfy the conjugation conditions

$$\begin{aligned} u_1^-(x, z) - u_2^+(x, z) = g(x, z), \\ p_1 \partial_\nu u_1^-(x, z) - p_2 \partial_\nu u_2^+(x, z) = h(x, z), \end{aligned} \quad (x, z) \in \Gamma, \tag{2}$$

where $g(x, z) \in C^{1,\beta}(\Gamma)$, $h(x, z) \in C^{0,\beta}(\Gamma)$ $(0 < \beta \leqslant 1)$ are given functions;

*E-mail: Evgeny.Lipachev@ksu.ru.