Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41, No. 7, pp.28–35, 1997 *UDC 517.925.51*

ON THE GLOBAL STABILITY OF THE NONAUTONOMOUS SYSTEMS. I

A.A. Kosov

1. Introduction

One of the main directions of the development of the Lyapunov function (LF) method (see [1]) is in the widening of the class of functions applied for investigation of the stability. A principal meaning for this class belongs to the introduction of an LF with a derivative of constant sign (see [2], [3]) and a Lyapunov vector-function (LVF) (see [4]). Within the frames of this direction, for different types of autonomous and periodic equations there were proved theorems on the stability and the asymptotic stability, which were based on the use of the Lyapunov functions of constant signs (LFCSs) (see [5]–[9]). As it was mentioned in [9] (p. 76), "a generalization of theorems to nonautonomous systems invokes serious difficulties". Nevertheless, for separate classes of nonautonomous equations, there were suggested ways for analyzing the stability on the base of LFCSs (see [10]–[14]).

In the present first part of the article we prove with the use of LFs of constant signs and LVFs some theorems on global asymptotic stability of nonautonomous systems with a Lipschitz right side. These theorems generalize to the nonautonomous case the corresponding results for autonomous and periodic systems (see [5]–[9]), as well as theorems on asymptotic stability for nonautonomous systems with LFs, which have a derivative of constant sign (see [15]). The technique of proof (see [12], [14]) essentially differs from both [5]–[9] and approaches, earlier suggested for nonautonomous case (see [10], [11]) and is based on consideration of a family of limit equations (see [16], [17]).

In the second part of the article, on a base of LFCSs the problem of global stability of a nonautonomous system of the second order is considered, for which the generalized Raus-Hurwitz conditions are fulfilled. For the comparison with the known results we shall consider examples, among these an independently interesting problem on estimation of the domain of global stability in the space of parameters for an aircraft in the regime of moving with large attack angles (see [18]).

2. The main results

We shall denote by $\mathrm{Lip}(n,m)$ the class of all continuous functions $F: R_+ \times R^n \to R^m$ which possess the property $F(t,0) \equiv 0$ and satisfy in every finite domain $S(0,r) = \{x \in R^n : \|x\| \le r < +\infty\}$ the Lipschitz condition with respect to x with the constant $L_F(r) > 0$ independent of the time t. Consider a system of differential equations

$$\dot{x} = X(t,x), \quad X(t,x) \in \mathrm{Lip}(n,n). \tag{1}$$

Supported by the Russian Foundation for Basic Research (project 97-01-01497).