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PHASE SPACE OF THE CAUCHY–DIRICHLET PROBLEM FOR THE OSKOLKOV EQUATION OF NONLINEAR FILTRATION

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a boundary $\partial\Omega$ of the class C^∞. In the domain $\Omega \times \mathbb{R}$, the following Cauchy–Dirichlet problem is considered:

$$u(x,0) = u_0(x), \quad x \in \Omega; \tag{0.1}$$

$$u(x,t) = 0, \quad (x,t) \in \partial\Omega \times \mathbb{R} \tag{0.2}$$

for the nonclassic equation in partial derivatives

$$(1 - \varkappa\Delta)u_t = \nu\Delta u - |u|^{q-2}u + f, \quad q \geq 2. \tag{0.3}$$

Equation (0.3) describes the dynamics of pressure of the Newton fluid, which is filtered in the porous medium (see [1]). In [2], problem (0.1)–(0.3) was considered under the condition of positivity of the coefficients $\varkappa$ and ν, which are responsible for elasticity and viscosity properties of the fluid, respectively. But it was shown in [3] that negative values of the parameter $\varkappa$ do not contradicts the physical sense. Therefore, the goal of this article is the investigation of the solvability of problem (0.1)–(0.3) under the conditions $\varkappa \in \mathbb{R} \setminus \{0\}$ and $\nu \in \mathbb{R}_+$.

Since for $\varkappa \in \mathbb{R}_-$ the operator attributed to the derivative on t in equation (0.3), generally speaking, is not positive definite, we have to use some other methods, unlike those in [2]. Our approach consists in reduction of problem (0.1)–(0.3) to the Cauchy problem

$$u(0) = u_0 \tag{0.4}$$

for semilinear differential operator equation of a Sobolev type (see [4])

$$L\dot{u} = Mu + N(u), \tag{0.5}$$

where the operator L may not be continuously reversible, in particular, so is its kernel $\ker L \neq \{0\}$. Further, using the theoretical results on σ-bounded operators (see [5]), we split equation (0.5) into singular and regular components, which will be investigated separately. The main goal of the work is in the investigation of the *morphology* (i. e., the structure, construction, and arrangement) of the phase space of equation (0.5) (see [6]).

The work consists of four Sections. In the first and second Sections, some known results from [5] and [7], which are necessary in the sequel, are given. In the third Section, the reduction of problem (0.1)–(0.3) to problem (0.4), (0.5) is described. In the last Section, the phase space is investigated.

Investigations are carried out in real Banach spaces, but when the "spectral" questions are discussed, the natural complex analogs of these spaces are introduced. All contours are oriented counter-clockwise and bound the domains that lie at the left with respect to this motion. The

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