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CONFORMALLY CONNECTED SPACE

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The theory of projectively connected spaces, as well as the theory of conformal spaces and submanifolds of conformal spaces, is sufficiently developed now. However the conformally connected spaces and submanifolds of these spaces remain little investigated. The only exception is the paper [1], where E. Cartan initiated investigation of differential geometry of conformally connected spaces, unfortunately his ideas have not gained development (see, e. g., the survey [2]).

In the present paper we find necessary and sufficient condition for a projectively connected space $P_{n,n+1}$ endowed with an invariant field of local Darboux hyperquadrics $Q_n^2 \subset P_{n,n+1}$ to be isomorphic to a conformally connected space $C_{n,n}$. Also we study intrinsic geometry of the normalized space $C_{n,n}$.

From now on we agree on the following index ranges: $\lambda,\mu,\rho = \overline{0,n+1}$; $i,j,k,l,s,t = \overline{1,n}$; $I,K,L = \overline{1,n+1}$; $A,B = \overline{n+1,n+N}$; $u,v,w = \overline{1,r}$.

1. Preliminary

Let us consider a fibered manifold $\mathfrak{M}$ with n-dimensional base B_n, N-dimensional fiber E_N, and r-dimensional Lie group G_r. By the Cartan-Laptev theorem [3], [4], in the fibered manifold $\mathfrak{M}$ a system of Pfaff forms ω^u establishes a fundamental group connection with structure group G_r determined by the invariant forms ω^u if and only if ω^u satisfy the structure equations

$$D\omega^u = \frac{1}{2}c^u_{vw}\omega^v \wedge \omega^w + \frac{1}{2}R^u_{ij}\theta^i \wedge \theta^j, \tag{1}$$

where $c^u_{vw} = -c^u_{wv} = \text{const}$, $R^u_{ij} = -R^u_{ji}$, $D\theta^i = \theta^j \wedge \theta^i_j$, $\theta^i = a^i_j du^j$ are base Pfaff forms on B_n, and u^i are coordinates of points of B_n.

If $x^A(u)$ are fiber coordinates of points of the fiber $E_N(u)$, then the map $\psi : E_N(u+du) \to E_N(u)$ which determines connection, is written as follows [3]:

$$x^A(u+du) \xrightarrow{\psi} x^A(u,du) = x^A(u) - \xi^A_u(u)\omega^u(u,du) + \pi\varepsilon^A, \tag{2}$$

where $\lim\limits_{\pi\to 0}\varepsilon^A = 0$.

A field of geometrical object X^A of the space $\mathfrak{M}$ is said to be invariant with respect to the connection (see [3]) if the infinitesimal map (2) sends the local object of the fiber $E_N(u+du)$ to the local object of the fiber $E_N(u)$ [3]. By [3], a field of geometrical object X^A is invariant with respect to a connection if and only if the differential equation system determining this object is written as follows:

$$dX^A = \xi^A_u(X)\omega^u, \tag{3}$$

where $\xi^A_u(X)$ is the system of basic function determining the object X^A, and ω^u are the connection forms of the space.