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One Projection Method for Linear Third Order Equation

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Abstract—In this paper we study the Galyorkin method with a special basis for a linear operator-differential equation of the third order in a separable Hilbert space. The projection method is based on the eigenvectors of the operator similar to the leading operator of equation. We obtain estimates for the convergence rate of approximate solutions in uniform topology.

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1. Introduction. At the present time there exists a great series of papers dedicated to the investigation of operator-differential equations. Initial and boundary problems for these equations were investigated from rather different positions using the wide arsenal of diverse methods. Generally the used methods are adapted for a specific type of equations. From the point of view of the spectral theory of linear operators in a Hilbert space and the Fourier method of investigation for operator-differential equations of the first and higher orders are presented in monograph [1] (pp. 131–156).

Amongst operator-differential equations, the first and second order equations are the most thoroughly studied ones. In this connection we can indicate papers [2]–[4], [2–4] dedicated to the investigation of strong solutions to the Cauchy problem for linear and nonlinear first order operator-differential equations. The existence, the uniqueness, and the continuous dependence of strong solutions to the Cauchy problem for different second order linear equations with variable domains have been proved in papers [5, 6].

The boundary-value problem for operator-differential equations with variable coefficients of even order was considered in paper [7], in which one has obtained sufficient conditions of existence of the unique regular solution. In paper [8] one has studied questions about classification of operator-differential equations of arbitrary order, problem definitions for these equations and their solvability in the Sobolev–Slobodetskii spaces. In papers [9, 10] one has investigated the boundary problem for the operator-differential equation of higher order with linear closed operators. Based on the proved by the author energetic inequality the solvability in energetic spaces has been established. The resolvability in the weak sense of the boundary problem for the third-order linear operator-differential equation with the variable domain of definition of equation operator has been established in [11]. In [12] one has found sufficient conditions of the regular resolvability of initial boundary-value problems for some class of the third order operator-differential equations with variable coefficients on the semiaxis and given exact estimates of the norms of operators of auxiliary derivatives through the main part of equation. Sufficient conditions on the constant coefficients of the third order operator-differential equation, which provide the regular resolvability of the boundary-value problem, are adduced in [13]. Thus one assume that the normal operator of equation has the inverse completely continuous one.

In the present paper we consider the boundary-value problem for the third order operator-differential equation with the main self-conjugate operator and its subordinated variable operator. The investigated projection method is constructed by proper elements of the operator B, similar to the main operator A. Generally speaking, operator A also has the complete orthogonal system of proper elements. However, in many cases the operator B has a simpler structure, which allows to write proper elements explicitly.

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