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GAKHOV EQUATION FOR OUTER MIXED BOUNDARY VALUE PROBLEM BY PARAMETER x

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Introduction

The internal problem corresponding to that considered in this article was posed first by V.N. Monakhov (see [1]). In [2] the solvability of internal problem on Riemann surfaces was proved, the character of non-uniqueness of solution, that depends on the geometry of the known part of boundary was investigated (for the history of issue, see detail in [1], [2]).

In this article we consider the case where the unknown domain D_z in the plane z contains the infinite point. In case the known part of the boundary of the sought domain is polygonal, an integral representation of the solution $z(\zeta)$, $\zeta \in D_\zeta$, is obtained, where D_ζ is the upper halfplane in the ζ-plane. As in the internal problem, in the proof of solvability the problem of determination of accessory parameters arises; however, here this difficult and interesting problem is not investigated. The main attention is paid to the univalence of the function $z(\zeta)$, which is defined by the resulting integral representation. A necessary and sufficient conditions for such a univalence is equality to zero of the residue of the derivative of the function $dz(\zeta)/d\zeta$ at the point ζ_0, which corresponds to the infinity in the plane z.

This equality can be treated as an equation in the variable ζ_0. In the outer inverse boundary value problem by parameter s the corresponding equation was first obtained by F.D. Gakhov (see [4]) who proved its solvability. Later this Gakhov equation and its generalizations were considered by many authors (see, e. g., [3]–[5]) who studied the uniqueness of the solution of this equation, the structure of the set of its roots, etc. In particular, in [6] it was proved that the Gakhov equation in the outer inverse boundary value problem by parameter s has a finite number of solutions.

The basic result of this article in the proof of solvability of an analog of the Gakhov equation for the outer mixed inverse boundary value problem by parameter x in the case of polygonal known part of boundary. In addition, with the use of results in the theory of polyanalytic functions we investigate the local structure of the set of solutions of this equation.

1. Statement of problem

Let D_z be a simply connected Jordan domain on the Riemann sphere, containing inside itself ∞, with the boundary L_z which consists of the known arc L_z^1 and a desired arc L_z^2. In what follows we assume that L_z^1 is a polygon with the vertices $z_k = x_k + iy_k$, $k = 1, \ldots, n$. Next, L_z^2 is assumed to be such that any straight line parallel to the imaginary axis, intersects it in at most one point and $x_1 = a < b = x_n$ (Fig. 1).

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