

ON FINITE GROUPS WHOSE PRINCIPAL FACTORS ARE SIMPLE GROUPS

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At the present time, when classification of finite simple groups is considered to be finished, the problem of studies of finite compound groups turns to be actual. V.A. Vedernikov introduced in [1] a number of classes of compound groups, among these — the class of c -supersoluble groups. A group is said to be c -supersoluble if it possesses the principal series with all its factors being simple groups. The class of all c -supersoluble groups is denoted by \mathfrak{U}_c . One can easily notice that both supersoluble and quasinilpotent groups are c -supersoluble. However these examples do not exhaust the whole class \mathfrak{U}_c . In [1] some properties of the class of all c -supersoluble groups were found, in particular, it was noted that \mathfrak{U}_c forms an S_n -closed formation.

In the present article we study other properties of the formation of all c -supersoluble groups. It is well-known (see, e.g., [2], Chap. VI) that the formation of all supersoluble groups is saturated, i.e., if the factor group $G/\Phi(G)$ is supersoluble, then so is the group G . The following example shows that the formation \mathfrak{U}_c is not saturated.

Example. Let $G = A_5$ be a group of order five with altering sign, K be a field composed by three elements. We denote by $A = A_K(G)$ the Frattini KG -module (see [3]). In view of [3], A is an irreducible KG -module of the dimension four. By the known Gaschütz theorem, there exists the Frattini extension $A \rightarrow E \twoheadrightarrow G$ such that $A \xrightarrow{G} \Phi(E)$ and $E/\Phi(E) \cong G$. Obviously, $E/\Phi(E) \cong G$ is c -supersoluble, but from the characteristics of module A cited above there follows that E is not c -supersoluble. Consequently, the formation \mathfrak{U}_c is saturated and therefore is non-local.

At the same time, for \mathfrak{U}_c , a close to saturation property can be derived from the following result.

Theorem 1. *A formation \mathfrak{U}_c is a compound one and has a maximal inner composition screen h such that*

$$h(N) = \begin{cases} \mathfrak{N}_p\mathfrak{A}(p-1), & \text{if } N \text{ is a simple } p\text{-group;} \\ \mathfrak{U}_c, & \text{if } N \text{ is a simple non-Abelian group.} \end{cases}$$

Hence with regard for theorem 4.12 of [4] (Chap. IV) there takes place

Corollary. If N is a normal soluble subgroup of the group G and $G/\Phi(N)$ is c -supersoluble, then G is c -supersoluble.

Note that formation \mathfrak{U}_c is not radical, because otherwise the group which is the product of normal supersoluble subgroups always were supersoluble, which is not true. On the other hand, it is known that the group $G = HK$, where H and K are normal supersoluble subgroups, is supersoluble if $(|G:H|, |G:K|) = 1$ (see, e.g., [5]), or if G has a nilpotent commutant (see [6]). In this direction we obtained the following

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