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Stabilization of the Second-Order Linear Time-Invariant Control Systems by a Delayed Feedback

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Abstract—We consider a static stabilization problem for a two-dimensional linear time-invariant control system with a delayed feedback. We obtain the necessary and sufficient conditions for the stabilizability of the system under consideration. The theorems proved in this paper show that such a delayed feedback approach is efficient in stabilizing the second-order linear systems.

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Key words and phrases: *linear time-invariant system, delayed feedback, stationary stabilization, asymptotic stability.*

1. INTRODUCTION

Stabilization of dynamic systems is one of the most important problems in the control theory. The interest to stabilization problems is stipulated both by the needs of the practice and by the existence of unsolved problems. Most efficient are methods and algorithms for stabilizing time-invariant systems. They are studied in many papers, see, for example, reviews in [1–6].

In the recent decades stabilization problems were studied very intensively. One of problems considered in many papers is the Brockett problem [4] on the stabilization of linear time-invariant systems with the help of a linear nonstationary feedback. The Brockett problem is solved in several important for practice works, namely, in papers [7, 8] by G. A. Leonov and [9] by L. Moreau and D. Aeyels. In these papers one constructs algorithms for the low- and high-frequency stabilization of linear time-invariant systems. For two- and three-dimensional time-invariant systems one proves that a nonstationary feedback extends the possibilities of the time-invariant stabilization.

When studying the existence of stabilization methods different from nonstationary ones for linear time-invariant systems (that extend the possibilities of the time-invariant stabilization) one encounters the following problem stated by G. A. Leonov.

Problem. *Is it possible to stabilize a linear time-invariant system by introducing a delay in the feedback? How does a linear time-invariant feedback with a delay affect the abilities to stabilize linear time-invariant systems?*

Note that the stabilization of systems with the help of a feedback is very important for practice. As is shown in the fundamental works of K. Pyragas [10, 11] and in the subsequent papers (for example,), this approach allows one to control chaos in various physical and chemical systems, in particular, in electronic chaotic oscillators and laser systems. In the mentioned papers one controls chaos by introducing a delayed feedback for stabilizing a nonstationary periodic orbit that belongs to a strange attractor. The system of the first approximation for nonlinear systems with respect to the periodic orbit is a linear system with a periodic matrix; it is well-known that it is reducible to a linear system with constant coefficients. According to the Lyapunov theorem, the asymptotic stability of the latter implies the asymptotic stability of the periodic solution to the corresponding nonlinear system. Therefore, in this case we also obtain the problem of stabilizing a linear time-invariant system with the help of a feedback

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