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EXISTENCE OF SOLUTIONS FOR DIFFERENTIAL INCLUSIONS WITH PARTIAL DERIVATIVES OF FRACTIONAL ORDER

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The Darboux problem for hyperbolic differential equations with multivalued right side (differential inclusions) was studied in [1]–[4] and by other authors.

In the present article we consider questions concerning the existence and some properties of solutions of the Darboux problem for differential inclusions with partial derivatives of fractional order.

Basic postulates of the theory of fractional integro-differentiation, its applications in the theory of integral and differential equations, as well as rich bibliography of works can be found in the monograph [5].

1°. We introduce the following notation: E^n is the n-dimensional Euclidean space with the norm $\|\cdot\|$ and the zero element θ; $C(P)$, $AC(P)$, $L(P)$, and $L_\infty(P)$ are, respectively, the space of continuous, absolute summable, measurable by Lebesgue, and bounded in essential functions which are defined in a domain P; $\operatorname{comp} E^n$ ($\operatorname{conv} E^n$) is the set of all nonempty compact (convex and compact, respectively) subsets from E^n with the Hausdorff metric $\delta(\cdot,\cdot)$; $\rho(\cdot,\cdot)$ is the distance between a point and a set in E^n; $\overline{\operatorname{co}} A$ is the closure of convex hull of a set A; $\Gamma(\cdot)$ is the gamma-function. A function $f : P \to E^n$ is called a *selector* of multivalued mapping (m. m.) $F : P \to \operatorname{comp} E^n$ if $f(x) \in F(x)$ for $x \in P$.

Let $G = (0,a] \times (0,b]$, $\overline{G} = [0,a] \times [0,b]$, $0 < \alpha, \beta \le 1$, $f(x,y) \in L(G)$. The expression

$$I_0^\alpha f(x,y) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(s,y)}{(x-s)^{1-\alpha}} ds, \quad x > 0, \tag{1}$$

is called (see [5]) the *partial left-side Riemann-Liouville integral of the fractional order* α with respect to variable x, and the expression

$$I_0^r f(x,y) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_0^x \int_0^y \frac{f(s,t) ds\, dt}{(x-s)^{1-\alpha}(y-t)^{1-\beta}}, \quad x > 0,\ y > 0, \tag{2}$$

— the *mixed left-side fractional Riemann-Liouville integral of the order* $r = (\alpha,\beta)$. If $f(x,y) \in L(G)$, then (see [5]) the integrals $I_0^\alpha f(x,y)$, $I_0^r f(x,y)$ are defined almost everywhere (a. e.) on G and belong to the space $L(G)$.

Let $f_{1-\alpha}(x,y) = I_0^{1-\alpha} f(x,y)$, $f_{1-r}(x,y) = I_0^{1-r} f(x,y)$, and $1-r = (1-\alpha, 1-\beta)$. Then the functions

$$D_0^\alpha f(x,y) = D_x f_{1-\alpha}(x,y), \quad D_0^r f(x,y) = D_{xy}^2 f_{1-r}(x,y) \\ \Big(D_x = \frac{\partial}{\partial x}, \quad D_{xy}^2 = \frac{\partial^2}{\partial x\, \partial y}\Big) \tag{3}$$

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