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ON A CERTAIN KÖTHE SPACE DETERMINED BY A DIFFERENTIAL OPERATOR

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We consider on the strip $\Pi = \{(x,y) \in R^2 : |x| < \infty,\ |y| \le 1\}$ differential equation

$$\alpha \frac{\partial^4 u}{\partial x^4} + 2\beta \frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = 0 \quad (\alpha - \beta^2 > 0) \tag{1}$$

with the boundary value conditions

$$u(x,y)|_{y=\pm 1} = \left.\frac{\partial u}{\partial y}\right|_{y=\pm 1} = 0. \tag{2}$$

We denote by U the class of all functions $u(x,y) \in C^4(\Pi)$ satisfying equation (1) and boundary value conditions (2).

Let $r \in N$. We consider a function $g_r(x) \in C^4(-\infty,\infty)$ such that

$$g_r(x) = \begin{cases} 1, & |x| \le r+1; \\ 0, & |x| \ge r+2. \end{cases}$$

Then for any function $u(x,y) \in U$ the product $u_r(x,y) = u(x,y) g_r(x) \in C^4(\Pi)$ satisfies the equation

$$\alpha \frac{\partial^4 u_r}{\partial x^4} + 2\beta \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{\partial^4 u_r}{\partial y^4} = \varphi_r(x,y) \tag{3}$$

and the boundary value conditions

$$u_r(x,y)|_{y=\pm 1} = \left.\frac{\partial u_r}{\partial y}\right|_{y=\pm 1} = 0, \tag{4}$$

where the function

$$\varphi_r(x,y) = 4\left(\alpha \frac{\partial^3 u_r}{\partial x^3} + \beta \frac{\partial^3 u_r}{\partial x \partial y^2}\right) g_r' + 2\left(3\alpha \frac{\partial^2 u_r}{\partial x^2} + \beta \frac{\partial^2 u_r}{\partial y^2}\right) g_r'' + 4 \frac{\partial u_r}{\partial x} g_r''' + \alpha u_r g_r^{\mathrm{IV}}$$

vanishes outside intervals $r+1 < |x| < r+2$.

By means of the Fourier transformation

$$u_r^*(y,\lambda) = \int_{-\infty}^{\infty} u_r(x,y) e^{-i\lambda x}\, dx, \quad \varphi_r^*(y,\lambda) = \int_{-\infty}^{\infty} \varphi_r(x,y) e^{-i\lambda x}\, dx,$$

the boundary value problem (3)–(4) turns into the following one:

$$\frac{d^4 u_r^*}{dy^4} - 2\beta\lambda^2 \frac{d^2 u_r^*}{dy^2} + \alpha\lambda^4 u_r^* = \varphi_r^*, \tag{5}$$