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ON INTEGRAL EQUATIONS FOR RIEMANN FUNCTION

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1. It is known (see [1], p. 199) that the Riemann function of the equation

$$u_{xy} + au_x + bu_y + cu = 0 \tag{1}$$

is a solution of the conjugated equation

$$v_{xy} - (av)_x - (bv)_y + cv = 0, \tag{2}$$

satisfying the conditions

$$\begin{aligned} v(x,\tau,t,\tau) &= \exp\int_t^x b(\xi,\tau)d\xi, \\ v(t,y,t,\tau) &= \exp\int_\tau^y a(\tau,\eta)d\eta. \end{aligned} \tag{3}$$

It is supposed that $a, b, c \in C^2$ in the domain under consideration and there exist $a_x, b_y \in C^2$. In order to determine v we rewrite (2) into the form

$$\Big(\frac{\partial}{\partial y} - a\Big)\Big(\frac{\partial}{\partial x} - b\Big)v = hv, \tag{4}$$

where h is Riemann's invariant (see [1], p. 176)

$$h = a_x + ab - c. \tag{5}$$

First let us construct the Riemann function R for (4) under assumption that $h \equiv 0$. It is easy to verify immediately that R is a solution of the problem

$$\Big(\frac{\partial}{\partial x} + b\Big)\Big(\frac{\partial}{\partial y} + a\Big)R = 0, \tag{6}$$

$$\begin{aligned} R(x,\tau,t,\tau) &= \exp\int_x^t b(\xi,\tau)d\xi, \\ R(t,y,t,\tau) &= \exp\int_y^\tau a(t,\eta)d\eta. \end{aligned} \tag{7}$$

Equation (6) is resolvable in quadratures (see [1], p. 177). We resolve it and obtain with regard to (7) that

$$R(x,y,t,\tau) = \exp\Big[\int_y^\tau a(x,\eta)d\eta + \int_x^t b(\xi,\tau)d\xi\Big]. \tag{8}$$

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