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CONVERGENCE OF WEIGHTED SUMS OF RANDOM ELEMENTS IN BANACH SPACES OF TYPE p

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1. Introduction

In the present article the notion of f sup-convergence of random elements, which is stronger than almost sure convergence, is introduced, the laws of large numbers with respect to this convergence for weighted sums of random elements with values in the Banach spaces of type p are studied.

A convergence, more strong than a. s. one, is defined as follows: for a continuous increasing function $f : \mathbf{R}^+ \to \mathbf{R}^+$, $f(0) = 0$, $f(\infty) = \infty$ and random elements $(T_n)_1^\infty$ which take values in the Banach space E the sequence $T_n \to 0$ in sense f sup if $\mathbf{E}f(\sup_{k\geq n} \|T_k\|) \to 0$ as $n \to \infty$. Obviouly, the f sup-convergence is a more strong one than both convergence a. s. and convergence in Orlicz space $L_f(E)$ (if only f satisfies the Δ_2-condition, see [1], theorem 9.4).

In the present article we shall find the moment conditions on the norms of random elements with their values in a space of type p by making the weighted sums of these elements to converge to zero in the sense f sup.

Let us introduce notation. In the sequel, E is always a real separable Banach space, $(X_k)_1^\infty$ is a sequence of independent symmetric random elements with values in E, and the symbol C stands for a finite positive constant which is not necessarily the same in each appearance. A sequence of random elements (X_k) is *stochastically dominated* by a positive random variable ξ (we write $(X_k) \prec \xi$) if there exists $C > 0$ such that for all $t > 0$

$$\sup_{k\geq 1} \mathbf{P}\{\|X_k\| > t\} \leq C\mathbf{P}\{\xi > t\}.$$

The notions of both Rademacher and stable types of Banach spaces, used in the sequel, can be found in [2] (Chap. 3–4). We recall also

Kwapien's inequality (see [3]). *Let $(X_k)_1^n$ be a sequence of independent symmetric random elements and $(s_k)_1^n \subset \mathbf{R}$. Then, for all $t \geq 0$, the probability satisfies the relation:*

$$\mathbf{P}\Big\{\Big\|\sum_{k=1}^n s_k X_k\Big\| > t\Big\} \leq 2\mathbf{P}\Big\{\max_{1\leq k\leq n} |s_k| \Big\|\sum_{k=1}^n X_k\Big\| > t\Big\}.$$

As for the history of the question, the study of the convergence of weighted sums is a natural extension of the studies of the Marcinkiewicz law of large numbers. The Marcinkiewicz law of large numbers was examined by T.A. Azlarov and N.A. Volodin [4], A. de Acosta [5] in Banach spaces of Rademacher type, by M. Marcus and W.A. Woyczynsky [6] in Banach spaces of stable type. Generalizations of these results to weighted sums of random elements were studied by A. Adler, A. Rosalsky and R.L. Taylor [7], Th. Mikosch and R. Norvaisa [8], R.L. Taylor [9]. The present article continues these investigations. Some results were announced in [10] and [11].