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On a Weighted Boundary-Value Problem in an Infinite Half-Strip for a Biaxisymmetric Helmholtz Equation

A. A. Abashkin[1*]

[1] *Samara State University of Architecture and Civil Engineering, ul. Molodogvardeiskaya 194, Samara, 443001 Russia*

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Abstract—We study a boundary-value problem for a generalized biaxisymmetric Helmholtz equation. Boundary conditions in this problem depend on equation parameters. By the method of separation of variables, using the Fourier–Bessel series expansion and the Hankel transform, we prove the unique solvability of the problem and establish explicit formulas for its solution.

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1. THE PROBLEM

For the equation

$$H^{\lambda}_{\mu,p}u = u_{xx} + u_{yy} + \frac{2\mu}{x}u_x + \frac{2p}{y}u_y + \lambda^2 u = 0, \quad p > 0, \quad \lambda > 0, \tag{1}$$

we study a boundary-value problem in the half-strip $D = \{(x, y) \mid 0 < x < a, 0 < y < \infty\}$.

The problem. Find a function $u(x, y)$ satisfying the condition

$$H^{\lambda}_{\mu,p}u(x, y) \equiv 0, \quad u(a, y) = q_2(y), \quad \lim_{y\to\infty} u(x, y) = 0 \quad \text{with} \quad x \in (0, 1), \tag{2}$$

and one of the following sets of conditions:

1) with $\mu, p \geq 1/2$,

$$u(x, y) \in C((0, a] \times (0, \infty)) \cap C^2(D), \tag{3}$$

$$\lim_{y\to 0+} y^{2p-1}u(x, y) = \varphi(x), \quad x \in (0, a], \quad p > 1/2, \tag{4}$$

$$\lim_{y\to 0+} \frac{u(x, y)}{\ln y} = \varphi(x), \quad x \in (0, a], \quad p = 1/2, \tag{5}$$

$$\lim_{x\to 0+} x^{2\mu-1}u(x, y) = q_1(y), \quad y \in (0, \infty), \quad \mu > 1/2, \tag{6}$$

$$\lim_{x\to 0+} \frac{u(x, y)}{\ln x} = q_1(y), \quad y \in (0, \infty), \quad \mu = 1/2, \tag{7}$$

2) with $\mu \geq 1/2$, $p < 1/2$,

$$u(x, y) \in C((0, a] \times [0, \infty)) \cap C^2(D), \tag{8}$$

$$u(x, 0) = \varphi(x), \quad x \in (0, a], \tag{9}$$

$$\lim_{x\to 0+} x^{2\mu-1}u(x, y) = q_1(y), \quad y \in [0, \infty), \quad \mu > 1/2, \tag{10}$$

*E-mail: samcocaa@rambler.ru.