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ON EXTENSION OF STOCHASTIC CONSTRAINTS IN THE CLASS OF FINITE-ADDITIVE PROBABILITIES

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Unstable problems of prediction in the class of mathematical expectation (ME) under moment constraints imposed on the choice of probability density and their abstract analogs are studied. A correct extension, universal in a wide class of perturbations of the system of conditions, in the class of finite-additive (f.-a.) probabilities with the property of weak absolute continuity with respect to a fixed measure is constructed. "Unilateral" stability of the generalized problem of prediction under non-monotone perturbations (NP) of inclusions which define stochastic constraints (SC) is established. A regularization of the attraction sets (in the class of approximate solutions-trends), which are realized in the conditions of NP and simultaneous weakening of SC, is constructed. We consider variants of SC which arise in the problems of treatment of statistic information, for which NP can carry the sense of deviations of the empiric means from the true ME.

1. Discussion of the problem

In the present article we consider an abstract analog of the problem of attainability "in mean" under fulfillment of SC conditions; we investigate the questions of sensitivity to perturbations of parameters of the mentioned SC. The latter can be defined via the data of statistic treatment of the measurement results in the form of empiric distributions or, in a more simple case, in the form of empiric mean-values. In the case, the true probability distribution is unknown. However, the information extracted from "intsructing" samples enables us to introduce a specific inverse problem. For the latter, there is characteristic a multivalence of the mapping which defines solution, as well as an essential instability. This motivates the application of regularizing procedures with the use of a priori given information (see in this connection, in particular, the studies [1]–[4]). However, such an information is not always attainable and may concern only separate fragments from the input collection of data. Thus, for example, in determining the empiric distribution along the sample of a discrete random value we operate with the relative frequencies but not with the events probabilities; the error of determination of the mentioned probabilities, even in case of the representative samples, cannot principally be eliminated. Here we should also add the errors of both computing and interpretations of various kinds. There are many cases where one fails in justified estimation of mentioned errors. In these conditions, it is required to use another (with respect to regularizations in the theory of ill-posed problems) technique (in this connection, see investigation on robust stochastics [5], [6]). In particular, their characteristic feature is in applying the infinitesimal analysis with the use of asymptotic analogs of intrinsic characteristics of random values (see [6], p.115). The investigation to be carried below is also oriented to the providing "correct" (in a certain sense) asymptotic effects. However, the basement here is not the "direct" passage to limit, but a certain compactification of the space of solutions. The below-used machinery is naturally related to the extensions of some functional-differential equations and their abstract

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