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MG-Deformations of a Surface of Positive Gaussian Curvature Under Assignment of Variation of Any Tensor Along an Edge

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Abstract—We investigate the infinitesimal MG-deformations of a simply connected surface with positive Gaussian curvature. We choose any symmetric tensor on the surface, variation of the first and the second invariant of this tensor equals given function along a boundary. The study of these boundary-value problems is reduced to the investigation of a solvability of Riemann–Hilbert boundary-value problem and to calculation of its index. As a result we get theorems of existence and uniqueness for the infinitesimal MG-deformation.

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INTRODUCTION

The study of surfaces infinitesimal deformations is one of research directions in the modern geometry. Historically, the first type of deformation studied was bending, i.e., the deformations that preserve the arc lengths on the surface. At the moment, the bendings are well-studied, therefore, we study various types of deformations different from bendings [1].

One of the most important questions in the theory of deformations is the solution of their existence and uniqueness problem. This question is especially interesting for surfaces with boundary. We then impose various additional conditions on the surfaces edges. For example, we have the following geometric type conditions: zero variations of the first quadratic form, normal curvature, mean curvature, and so on.

Here we study infinitesimal MG-deformations. The spherical image of the surface is pointwise preserved and the condition $\delta K = \sigma$ is met, where δK is the Gaussian curvature variation, σ is a given function of class $D_{1,p}$, $p > 2$, on the surface under these deformations. The author introduced and considered this type of deformation in [2].

Any symmetric tensor on the surface has two invariants. We can specify variations of these invariants along the surface edge under deformation and obtain a variety of different boundary conditions.

In this paper, we prove the existence and uniqueness theorems for infinitesimal MG-deformations for the given variations of the first or second invariant of any symmetric surface tensor along the boundary of the surface. Here we apply the results and relations obtained earlier for infinitesimal MG-deformations for the vanishing variation of the mean curvature along the edge of the surface [3].

This paper consists of five Sections, starting with the introduction, then we formulate the main results and prove their consequences in the first Section; the second Section gives a detailed definition of infinitesimal MG-deformations; we describe the equations of these deformations in the third part of the paper. In the fourth Section, we study the boundary conditions and prove the theorems.

The results presented in the paper are of a theoretical nature.

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