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THE ACCURACY OF APPROXIMATE SYMMETRIES OF EQUATIONS DESCRIBING THE DYNAMICS OF THE NON-NEWTONIAN FLUID AND INVARIANT SOLUTIONS OF THESE EQUATIONS. I

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1. In [1] the approximate symmetries of the following equation were investigated in case that n is odd:

$$u_t = (u^2\sigma^n)_x + \varepsilon\Phi(t,x,u,\sigma). \quad \sigma = uu_x,$$

This equation describes the dynamics of the surface of the non-Newtonian fluid with the Ostwald–de Waele power rheological law and the mass balance, or the surface source, equaling to $\varepsilon\Phi$ (see [2]). (Hereafter we set $\sigma = uu_x$.) In particular, it was demonstrated that the equation

$$u_t = (u^2\sigma^n)_x + \varepsilon[\zeta_1'(t)xu - (\tfrac{1}{2}x^2\theta_1'(t) + x\theta_2'(t))\sigma u^{-1} + f(t,u,\sigma)] \tag{1.1}$$

admits, accurate to $O(\varepsilon^2)$, the approximate symmetry group with the following operator

$$X = \partial_x + \varepsilon X_1, \quad X_1 = \varphi\partial_t + (\theta_1 x + \theta_2)\partial_x + \zeta_1 u\partial_u + \zeta_2\sigma\partial_\sigma,$$

where φ, θ_1, θ_2 are arbitrary functions of time t, $f(t,u,\sigma)$ is an arbitrary function, $\zeta_1 = \frac{(n+1)\theta_1-\varphi'}{2n+1}$, and $\zeta_2 = \frac{\theta_1-2\varphi'}{2n+1}$.

The equation

$$u_t = (u^2\sigma^n)_x + \varepsilon\Big[\zeta_1 u - \Big(\theta_1 x + \theta_2 + \lambda\int\theta_1\,dt\Big)\sigma u^{-1} + f(I,u,\sigma)\Big], \quad I = x + \lambda t, \tag{1.2}$$

admits the approximate symmetry group with the operator

$$X_\lambda = \partial_t - \lambda\partial_x + \varepsilon X_1. \tag{1.3}$$

It is natural to pose the question about the conditions that provide not only approximate but exact invariance of equations (1.1), (1.2), as respects the groups having the operators X and X_λ, respectively. Besides, it is desirable to find the invariant solutions for the defined exactly invariant equations. In this paper, we consider these problems.

In the first part (Items 1–3) we find all exactly invariant equations of the form (1.1), (1.2) and ordinary differential equations for their invariant solutions. In the second part (Item 4) we investigate the invariant solutions with the use of the qualitative theory of dynamic systems. Also, we show how to apply the qualitative analysis of the invariant solutions for solving the boundary value problems for equations (1.1), (1.2).

For simplicity of notation, we set $p = \frac{n+2}{2n+1}$, $q = \frac{1}{2n+1}$, $r = \frac{2n+1}{5n+4}$.

2. Using the infinitesimal criterion of exact invariance (see [3], [4]), we can solve the problem of exact invariance of equation (1.1) as respects the group with the operator $X = \partial_x + \varepsilon X_1$.

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