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TO THE LOCAL CONTROLLABILITY IN THE CRITICAL CASE

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Introduction

In the present article we shall study conditions of controllability to zero of the system

$$\dot{x} = f_0(x) + uf_1(x), \quad (x,u) \in \mathbb{R}^n \times [-1,1] \tag{1}$$

in the critical case (i. e., where the system of linear approximation is not completely controllable). In the present article we introduce the concept of a stable local controllability and show that, if the system of linear approximation is completely controllable, then system (1) is stably controllable; but if the property of the complete controllability of a system of linear approximation is absent, then one of the following three cases is realized: 1) system (1) is stably locally controllable; 2) system (1) is controllable; 3) system (1) is uncontrollable. We separate a class of systems of the form (1), for which the stable local controllability takes place. It is shown that an N-controllability (see [1]) of the system

$$\dot{x} = f(x,u), \quad x \in \mathbb{R}^n, \quad u \in U \subset \mathbb{R}^m \tag{2}$$

implies the stable controllability. Thus, the property of the N-controllability is the most strong among today known properties of controllability, and the stable controllability occupies an intermediate position between the N-controllability and the local controllability.

A series of works [1]–[10] is dedicated to problems of investigation of controllability conditions of systems of the from (2); moreover, in [2], [7], and [10] sufficient conditions of controllability were obtained for the critical case. A complete answer to the question on the stable controllability was obtained only for systems of second order. For an arbitrary $n > 2$, a complete answer on the local controllability remains open. In some papers the verification of controllability in the critical case presupposes the presence of some objects, whose existence and construction seem to be very problematic (see [10]). A situation similar to the construction of Lyapunov's functions in the second Lyapunov method arises here; however, it does not make less important the dignity of such assertions.

1. Notation and definitions

In the present article we use the following notation: $\mathbb{R}^n$ is an Euclidean space of dimension n with norm $|x| = \sqrt{x^*x}$ ($*$ stands for transposition); $\mathbb{R} \doteq \mathbb{R}^1$; $\mathbb{R}^+ \doteq [0,\infty)$; $O^n_\varepsilon(x_0) \doteq \{x \in \mathbb{R}^n : |x - x_0| < \varepsilon\}$; $O^n_\varepsilon \doteq O^n_\varepsilon(0)$; $\operatorname{int} G$ is the interiority, $\overline{G}$ is the closure, ∂G is the boundary of a set $G \subset \mathbb{R}^n$; $C^n(\mathbb{R}^k,\mathbb{R}^p)$ is the space of n times continuously differentiable functions which act from $\mathbb{R}^k$ to $\mathbb{R}^p$.

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