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The Dirichlet Problem for a Mixed-Type Equation of the Second Kind in a Rectangular Domain

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1. Statement of the problem. Consider an equation of mixed type

$$Lu \equiv u_{xx} + \operatorname{sgn} y \cdot |y|^m u_{yy} - b^2 u = 0, \tag{1}$$

where $0 < m < 2$, $b = \text{const} \geq 0$, in a rectangular domain $D = \{(x,y) \mid 0 < x < 1,\ -\alpha < y < \beta\}$, α, β are given positive numbers.

Dirichlet problem. Find a function $u(x,y)$ defined on D which satisfies the conditions

$$u(x,y) \in C(\overline{D}) \cap C^1(D) \cap C^2(D_+ \cup D_-), \tag{2}$$

$$Lu(x,y) \equiv 0,\ \ (x,y) \in D_+ \cup D_-, \tag{3}$$

$$u(0,y) = u(1,y) = 0,\ \ -\alpha \leq y \leq \beta, \tag{4}$$

$$u(x,\beta) = f(x),\ \ 0 \leq x \leq 1, \tag{5}$$

$$u(x,-\alpha) = g(x),\ \ 0 \leq x \leq 1, \tag{6}$$

where f and g are given sufficiently smooth functions such that $f(0) = f(1) = g(0) = g(1) = 0$.

In [1] (p. 303), it was first shown that certain problems of transonic gas dynamics reduce to the Dirichlet problem for equations of mixed type. In [2], it was shown that the Dirichlet problem for the Lavrentiev equation $u_{xx} + \operatorname{sgn} y \cdot u_{yy} = 0$ is ill-posed. After this paper, the problem arose of finding mixed domains for which the Dirichlet problem is well-posed. Later the Dirichlet problem for equations of mixed type was studied by many researchers [3]–[11]. A detailed bibliography of papers devoted to this subject can be found in [11]. In these papers, the uniqueness of solution of the Dirichlet problem for equations of mixed type was proved with the use of the extremum principle or the method of integral identities, and the existence of a solution was proved by the method of integral equations or by separation of variables.

The Tricomi problem and other similar problems for mixed-type equations of the second kind were studied in [12]–[15]. In [11] (p. 33), the Dirichlet problem was considered for the equation

$$L(u) \equiv u_{xx} + \operatorname{sgn} y \cdot [|y|^m u_{yy} + a|y|^{m-1} u_y + b|y|^{m-2} u] = 0,\ \ 0 < m < 2,$$

with the following conjugation conditions:

$$\lim_{y \to +0} y^{-p_2} u = \lim_{y \to -0} (-y)^{-p_2} u,$$

$$\lim_{y \to +0} [y^{1-p_1} u_y - p_2 y^{-p_1} u] = \lim_{y \to -0} [(-y)^{1-p_1} u_y + p_2 (-y)^{-p_1} u],$$

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