

THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF ONE CLASS OF INTEGRAL EQUATIONS

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We consider the equation

$$x(t) = \int_0^t \sum_{j=1}^n K_j(t-s) a_j(s) x(s) ds + f(t), \quad (1)$$

where K_j , a_j , f are continuous on $[0, \infty)$ complex-valued functions.

If $a_j(t) = \text{const}$ and $K_j \in L_1[0, \infty)$, then the asymptotic of solutions of equation (1) is well known (see, for instance, [1]–[4]). For periodic functions $a_j(t)$, the asymptotic of solutions is studied in [5], [6]. In this paper, we consider the case where $a_j(t)$ have an asymptotical expansion in power scale in a neighborhood of $+\infty$:

$$a_j(t) \sim a_{j0} + \frac{a_{j1}}{t+1} + \cdots + \frac{a_{jk}}{(t+1)^k} + \cdots, \quad t \rightarrow \infty. \quad (2)$$

(We use the degrees $(t+1)^k$ instead of t^k on account of technical reasons only.)

Assume that for all sufficiently small $\delta > 0$ and for a certain μ ,

$$e^{-\delta t} K_j(t) \in L_1[0, \infty), \quad K_j(t) = O(e^{\mu t}), \quad f(t) = O(e^{\mu t}). \quad (3)$$

Let λ_l stand for the situated in the half-plane $\operatorname{Re} z \geq 0$ roots of the equation

$$\sum_{j=1}^n a_{j0} \widehat{K}_j(z) = 1, \quad (4)$$

where $\widehat{K}_j(z) = \int_0^\infty e^{-zt} K_j(t) dt$ is the Laplace transformation, and let $\lambda_1, \dots, \lambda_r$ be all the roots of equation (4) such that $\operatorname{Re} \lambda_k = \lambda = \max_l \operatorname{Re} \lambda_l$.

We put $A_k = \left\{ u(t) \in C[0, \infty) : u(t) = \sum_{j=0}^k \frac{u_j}{(t+1)^j} + o(\frac{1}{(t+1)^k}) \right\}$ and $A_\infty = \bigcap_{k=0}^\infty A_k$. Thus, relation (2) means that $a_j \in A_\infty$.

The main result of this paper is

Theorem. *Let assumptions (2) and (3) be fulfilled for $\mu < \lambda$ and let all the roots $\lambda_1, \dots, \lambda_r$ be simple. Then the solution $x(t)$ of equation (1) is representable in the form*

$$x(t) = \sum_{j=1}^r e^{\lambda_j t} (t+1)^{\alpha_j} b_j(t),$$

where α_j are certain complex values, and $b_j \in A_\infty$.

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