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THE CONTINUITY BY KUDRYAVTSEV OF QUASICONFORMAL MAPPINGS

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Along with the known continuity by Hölder, in considering a distortion of quasiconformal mappings $f : G \to E^n$ of an open domain $G \subset E^n$, $n \geq 3$, on hypersurfaces in G the notion of an $(n-1)$-continuity arises which pertains to the vector function

$$f(x) = [f_1(x), f_2(x), \dots, f_m(x)], \quad m \geq n-1,$$

and which degenerates for $m < n-1$. Since such a continuity becomes apparent only when the Kudryavtsev variation of a quasiconformal mapping is bounded, it seems to be intrinsic to call it the continuity by Kudryavtsev.

Here and in what follows E^n is an Euclidean space of points $x = (x_1, x_2, \dots, x_n)$, H_{n-1} is the Hausdorff measure in E^n of dimension $n-1$, $B_n(x,r)$ is a ball in E^n with center at the point x and of the radius r, $S_{n-1}(R)$ is the boundary of the ball $B_n(R) = B_n(0,R)$,

$$\sigma_{n-1} = H_{n-1}(S_{n-1}(1)),$$
$$L = \{x \in E^n : x_2 = x_3 = \cdots = x_{n-1} = 0\},$$
$$C = \{x \in L : x_1^2 + x_n^2 = 1\}, \quad \Phi(x) = \frac{(x_1, 0, \dots, 0, x_n)}{\sqrt{x_1^2 + x_n^2}} : E^n \setminus L \to C$$

is the composition of projection of the space E^n onto the $(n-2)$-axis L with the next radial projection of the subspace L onto the circle C, $P_t = B_n(R) \cap \Phi^{-1}(t)$ is a half-hyperball rotating under change of the parameter $t \in C$ with the rotation $(n-2)$-axis L.

For the homeomorphism $f : G \to E^n$ of Sobolev's class $W_n^1(G)$ and the number $p \geq 1$, we consider the Kudryavtsev p-variation (see [1]): $\overset{p}{\underset{G}{\bigvee}} f = \int_G |Jf(x)|^p dx$, where $Jf(x)$ is the Jacobian of f at the point $x \in G$, which exists almost everywhere in G.

A mapping $f : G \to E^n$ of class $W_n^1(G)$ is said to be q-quasiconformal ($q \geq 1$) if $|\nabla f(x)|^n \leq q|Jf(x)|$ for almost all $x \in G$, where $|\nabla f(x)|$ stands for the spectral norm of the Jacobi matrix of the mapping f at the point x. As follows from the general theory of quasiconformal mappings (see [2]), under their restriction to compact subdomains the Kudryavtsev p-variation is finite for values of parameter p which are sufficiently close to unit. The property mentioned in the title of the article becomes apparent as $p > 2(n-1)/n$.

Theorem 1. *If a quasiconformal homeomorphism $f : B \to E^n$ of the ball $B = B_n(R)$ has a bounded Kudryavtsev p-variation for $p = \frac{\alpha(n-1)}{n(\alpha-1)}$, $1 < \alpha < 2$, then the inequality of isoparametric type*

$$\inf_{t \in C} H_{n-1}(f(P_t)) \leq C(n, \alpha, q) R^{(n-\alpha)/\alpha} \Big(\overset{p}{\underset{B}{\bigvee}} f\Big)^{(\alpha-1)/\alpha}$$