

The Study of Boundary-Value Problems for a Singular B -Elliptic Equation by the Method of Potentials

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Received December 1, 2009

Abstract—In this paper we apply the method of potentials for studying the Dirichlet and Neumann boundary-value problems for a B -elliptic equation in the form

$$\Delta_{x''} u + B_{x_{p-1}} u + x_p^{-\alpha} \frac{\partial}{\partial x_p} \left(x_p^\alpha \frac{\partial u}{\partial x_p} \right) = 0,$$

where $\Delta_{x''} = \sum_{j=1}^{p-2} \frac{\partial^2}{\partial x_j^2}$, $B_{x_{p-1}} = \frac{\partial^2}{\partial x_{p-1}^2} + \frac{k}{x_{p-1}} \frac{\partial}{\partial x_{p-1}}$ is the Bessel operator, $0 < \alpha < 1$ and $k > 0$ are constants, $p \geq 3$. We prove the unique solvability of these problems.

DOI: 10.3103/S1066369X10050117

Key words and phrases: *Bessel operator, B -elliptic equation, Dirichlet problem, Neumann problem, method of potentials.*

Let E_p^{++} stand for the part $x_p > 0$, $x_{p-1} > 0$ of the p -dimensional Euclidean space of points $x = (x', x_p)$, $x' = (x'', x_{p-1})$, $x'' = (x_1, x_2, \dots, x_{p-2})$; let Ω be a finite domain in E_p^{++} bounded by a hypersurface Γ and parts Γ_0 and Γ_1 of hyperplanes $x_{p-1} = 0$ and $x_p = 0$, respectively; $\Omega_e = E_p^{++} \setminus (\Omega \cup \Gamma)$. We denote by $C_B^k(\cdot)$ the set of even with respect to x_{p-1} functions from the class $C^k(\cdot)$. We denote by $C_0(\Gamma)$ the set of functions $\varphi(x)$ from the class $C(\Gamma)$ such that

$$\varphi(x) = o(1) \text{ as } x_p \rightarrow 0.$$

In E_p^{++} we consider the following singular B -elliptic equation:

$$T_B[u(x)] = \Delta_{x''} u + B_{x_{p-1}} u + x_p^{-\alpha} \frac{\partial}{\partial x_p} \left(x_p^\alpha \frac{\partial u}{\partial x_p} \right) = 0, \quad (1)$$

where $\Delta_{x''} = \sum_{j=1}^{p-2} \frac{\partial^2}{\partial x_j^2}$, $B_{x_{p-1}} = \frac{\partial^2}{\partial x_{p-1}^2} + \frac{k}{x_{p-1}} \frac{\partial}{\partial x_{p-1}}$ is the Bessel operator, $0 < \alpha < 1$ and $k > 0$ are constants, $p \geq 3$.

Taking into account the results of paper [1], we construct a fundamental solution to Eq. (1) with a singularity at the point $(0', x_{p0})$. With the help of the generalized shift operator we obtain a fundamental solution to Eq. (1) with a singularity at an arbitrary point $x_0 \in E_p^{++}$. We show that with small values of $\rho_{xx_0}^2 = |x - x_0|^2$ the fundamental solution to Eq. (1) is representable in the form

$$\mathcal{E}(x; x_0) = a C_k \frac{\Gamma(2 - \alpha) \Gamma(\frac{k}{2}) \Gamma(\frac{p-2}{2})}{2^{\alpha+1} \Gamma(1 - \frac{\alpha}{2}) \Gamma(\frac{p-\alpha}{2})} (x_{p-1} x_{p-1,0})^{-\frac{k}{2}} (x_p x_{p0})^{-\frac{\alpha}{2}} \rho_{xx_0}^{2-p} + \mathcal{E}^*(x; x_0), \quad (2)$$

where $\mathcal{E}^*(x; x_0)$ is the regular part of the fundamental solution.

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