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THE LOGARITHMIC ESTIMATES OF CONVERGENCE RATE FOR CERTAIN METHODS OF SOLUTION OF THE INVERSE CAUCHY PROBLEM IN A BANACH SPACE

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1. A great deal of publications (see, for instance, [1]–[3] and their bibliography) treats estimations of convergence rate for various methods of solution of linear ill-posed operator equations

$$Bu = f, \quad u \in X. \tag{1}$$

Let us assume that B is a linear continuous operator which acts in a Banach space X and does not have a continuous inverse operator. Since problem (1) is ill-posed, we can obtain qualified estimates of convergence rate for approximate methods of its solution only under certain additional restrictions on the desired solution. As a rule, these restrictions are conditions of sourcewise representability of the initial discrepancy $u^* - \xi$. Here u^* is a solution of (1); ξ is some initial approximation of the method under consideration.

A lot of known methods of approximation of solutions of equations (1) can be obtained within the bounds of the general scheme ([3], p. 39)

$$u_\alpha = (E - \Theta(B,\alpha)B)\xi + \Theta(B,\alpha)f, \quad \alpha \in (0, \alpha_0], \tag{2}$$

which determines approximations u_α such that $\lim\limits_{\alpha\to 0} u_\alpha = u^*$. The function of the operator B in (2) is understood in the sense of the suitable operator calculus. If the operator B is sectorial, then the assumption of the power sourcewise representability $u^* - \xi \in R(B^p)$, $p > 0$, ensures the validity of the power estimate for the convergence rate $\|u_\alpha - u^*\| \le C_1\alpha^p \quad \forall \alpha \in (0, \alpha_0]$ with the same exponent p (see [3], p. 42) for an extensive class of generator functions $\Theta(\lambda, \alpha)$. Here and in what follows, $C_1, C_2, \dots$ stand for positive absolute constants. In typical cases, the sourcewise representability means the refined smoothness of the discrepancy $u^* - \xi$ as compared with the smoothness of the elements of the initial space X. For the space of summable functions $X = L_r$, $1 < r < \infty$, this restriction is often reduced to the condition that the initial discrepancy $u^* - \xi$ belongs to a certain Sobolev space ([3], p. 24). If X is a Hilbert space, $B^* = B \ge 0$, then the logarithmic sourcewise representability $u^* - \xi \in R((-\ln B)^{-p})$, $p > 0$, which implies the logarithmic estimate for the convergence rate

$$\|u_\alpha - u^*\| \le C_2(-\ln \alpha)^{-p} \quad \forall \alpha \in (0, \alpha_0] \tag{3}$$

(see [4]), has the same interpretation. It is interesting to analyze the conditions of sourcewise representability, which imply the logarithmic estimate of the convergence rate (3), for a Banach space X, too.

In this paper, we consider equation (1) with the operator $B = U(T)$, where $U(t)$, $0 \le t \le T$, is a semigroup of bounded operators acting in a Banach space X. We prove that the sourcewise representability condition $u^* - \xi \in R(A^{-p})$, $p > 0$, where $-A$ is a generator of the semigroup $U(t)$,