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A Criterion of Uniqueness of a Solution to the Dirichlet Problem with the Axial Symmetry for the Three-Dimensional Mixed Type Equation with the Bessel Operator

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Abstract—By the method of spectral expansions we establish a uniqueness criterion of a solution to the Dirichlet problem for the three-dimensional mixed-type equation with the Bessel operator.

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1. Problem definition. Let us consider the mixed type equation of the second kind

$$T_B u \equiv B_{x_1} u + \frac{\partial^2 u}{\partial x_2^2} + (\operatorname{sgn} x_3)|x_3|^m \frac{\partial^2 u}{\partial x_3^2} - a^2 u = 0 \tag{1}$$

in a semicylinder $D = \{(x_1, x_2, x_3) | x_1^2 + x_2^2 < 1,\ x_1 > 0,\ -\alpha < x_3 < \beta\}$, where $a > 0$, $0 < m < 2$, $\alpha > 0, \beta > 0$ are given real numbers, $B_{x_1} = \frac{\partial^2}{\partial x_1^2} + \frac{k}{x_1}\frac{\partial}{\partial x_1}$, $k > 0$, is the Bessel operator.

The easiest way to investigate questions about the correctness of boundary-value problems for Eq. (1) in a semicylinder D is the using of cylindrical coordinates (ρ, φ, z), which are connected with Cartesian coordinates by formulas $x_1 = \rho\cos\varphi$, $x_2 = \rho\sin\varphi$, $x_3 = z$, $(0 \le \varphi \le \pi)$.

Eq. (1) in cylindrical coordinates with the axis symmetry (with $\frac{\partial u}{\partial \varphi} = 0$) takes the form

$$T_B u \equiv \frac{\partial^2 u}{\partial \rho^2} + \frac{k+1}{\rho}\frac{\partial u}{\partial \rho} + (\operatorname{sgn} z)|z|^m \frac{\partial^2 u}{\partial z^2} - a^2 u = 0. \tag{2}$$

The Dirichlet problem with the axial symmetry. In the area D it is required to find a function $u(\rho, z)$, satisfying conditions

$$u(\rho, z) \in C^2(D^+ \cup D^-) \cap C(\overline{D}), \tag{3}$$

$$T_B u(\rho, z) \equiv 0,\ \ (\rho, z) \in D^+ \cup D^-, \tag{4}$$

$$\frac{\partial u}{\partial \rho}\Big|_{\rho=0} = 0, u|_{\rho=1} = 0,\ \ -\alpha \le z \le \beta, \tag{5}$$

$$u|_{z=-\alpha} = \psi(\rho), u|_{z=\beta} = \varphi(\rho),\ \ 0 \le \rho \le 1, \tag{6}$$

$$\lim_{z\to 0+0} u_z(\rho, z) = \lim_{z\to 0-0} u_z(\rho, z),\ \ 0 < \rho < 1,\ \ 0 < m < 1, \tag{7}$$

$$\lim_{z\to 0+0} z^{m-1} u_z = -\lim_{z\to 0-0} (-z)^{m-1} u_z,\ \ 0 < \rho < 1,\ \ 1 < m < 2, \tag{8}$$

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