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Optimal Control: Nonlocal Conditions, Computational Methods, and the Variational Principle of Maximum

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Abstract—This paper surveys theoretical results on the Pontryagin maximum principle (together with its conversion) and nonlocal optimality conditions based on the use of the Lyapunov-type functions (solutions to the Hamilton–Jacobi inequalities). We pay special attention to the conversion of the maximum principle to a sufficient condition for the global and strong minimum without assumptions of the linear convexity, normality, or controllability. We give the survey of computational methods for solving classical optimal control problems and describe nonstandard procedures of nonlocal improvement of admissible processes in linear and quadratic problems. Furthermore, we cite some recent results on the variational principle of maximum in hyperbolic control systems. This principle is the strongest first order necessary optimality condition; it implies the classical maximum principle as a consequence.

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INTRODUCTION

This paper surveys the results on the Pontryagin maximum principle (together with its conversion), nonlocal optimality conditions based on the use of the Lyapunov-type functions (solutions to Hamilton–Jacobi inequalities), nonstandard (and also nonlocal) numerical improvement methods as applied to classical optimal control problems in various statements. In addition, we adduce some recent results on the variational principle of maximum in hyperbolic control systems. This principle is the strongest (in the first order) necessary optimality condition that implies the classical maximum principle.

Naturally, we selected papers for the survey at our own discretion. We were most interested in the research of the Irkutsk school of the optimal control. Let us describe the considered subject in detail.

The maximum principle: necessity and sufficiency. It would be improper to avoid the mentioned theme in the year of the centenary celebration of Lev Semyonovich Pontryagin, a prominent mathematician of the XXth century, the academician who was first to establish the fundamental maximum principle (MP). Despite the huge amount of papers devoted to the theory of the MP for various classes of problems (published in recent years), here we consider only the almost classical problem with general endpoint constraints. For this problem, along with the well-known monograph [1] followed by [2], various (immaculately polished) proofs are described in [3]–[10]. However, in this paper we pay the most attention to the conversion of the MP into a sufficient condition for a global and strong minimum [11]–[13] without any assumptions of the linear convexity, normality, or controllability.

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