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Overlapping Iterated Function Systems on a Segment

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Abstract—Overlapping iterated function systems generate families of injective mappings from the attractor onto shift-invariant subsets of the code space. In this paper we consider an example of such a family for the uniformly linear systems of iterated functions on the unit segment.

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1. INTRODUCTION

At present, the study of iterated function systems (IFS) and their attractors is one of the main directions of research in fractal geometry. The code space that arises naturally in this connection serves a unified model for attractors of all IFS's with the same number of functions. The problem lies in the fact that, with exception of the trivial case when the attractor is homeomorphic to the Cantor set, the code mapping from the code space to the attractor is not injective. Thus, the necessity arises to construct sections of the code mapping. One of the possible methods of constructing sections is the use of top addresses [1–3].

In this paper, we construct sections using partitions of the attractor into subsets [4–6]. In Section 2, we formulate the definition of an IFS and its attractor. Section 3 is devoted to sections of the code mapping and their properties. Starting with Section 4, we consider a concrete example of IFS on the unit segment. We prove that sections preserve the order relation (Lemma 4). Section 5 is devoted to the study of the address space $\Omega_{t,p}$. The main results of this section are Theorem 1 which gives equivalent definitions of the space $\Omega_{t,p}$ and Theorem 2 which generalizes the result of Lemma 4. In Section 6, we prove Theorem 3 which gives an answer to the question: For which values of t and p does a fixed element of the code space ω belong to $\Omega_{t,p}$?

2. ITERATED FUNCTION SYSTEMS

Let $(\mathbb{X}, d)$ be a complete metric space, $N \in \mathbb{N}$ and $I := \{0, \dots, N\}$. Let $g_k : \mathbb{X} \to \mathbb{X}$, $k \in I$, be a collection of contraction mappings with contraction coefficients $c_k \in [0, 1)$, i.e., $d(g_k(x), g_k(y)) \leq c_k d(x, y)$ for any $x, y \in \mathbb{X}$ and $k \in I$. Denote by $\mathcal{K}(\mathbb{X})$ the set of nonempty compact subsets of $\mathbb{X}$. It is known that $\mathcal{K}(\mathbb{X})$ with the Hausdorff metric $d_{\mathcal{H}}$ is a complete metric space. The collection

$$\mathcal{G} := (\mathbb{X}, g_0, \dots, g_N)$$

is called an iterated function system (IFS) on $\mathbb{X}$. An IFS $\mathcal{G} = (\mathbb{X}, g_0, \dots, g_N)$ is called injective (open) if all mappings g_k are injective (open).

Define a mapping

$$G : \mathcal{K}(\mathbb{X}) \to \mathcal{K}(\mathbb{X}), \quad G(K) := \bigcup_{k=0}^{N} g_k(K).$$

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