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ON AN EXTREMAL PROBLEM OF THE WING THEORY

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In [1], a class of profiles was introduced which are flowed by ideal incompressible fluid and have a fixed exterior conformal radius equaling 1. On this class, for a given attack angle $\alpha/2$ and for a velocity constant at infinity, a problem is posed to determine the extremal profile, on which the minimum of the maximal velocities of the flow is attained for the whole class. This problem in [1] was reduced to a problem on the class Σ (see [2], p. 110) (including the functions $F = z + a_0 + \sum\limits_{n=1}^{\infty} a_n z^{-n}$, univalent in $D^- = \{|z| > 1\}$)

$$\sup_{|\zeta|>1} \left| \left(1 - \frac{1}{\zeta}\right)(1 + \frac{e^{i\alpha}}{\zeta})/F'(\zeta) \right| \to \min_{F\in\Sigma} = M(\alpha), \quad \alpha > 0. \tag{1}$$

In the present article, under quite plausible assumptions upon the extremals, a hypothesis from [1] that the extremal profile is a slit will be proved. We also present certain estimative results.

Lemma 1. *Suppose that F supplies the extremum in problem (1), $|F(\zeta)| \ge \delta_0$ and $F(|\zeta| = 1)$ is a continuous curve having a curvature for all $\zeta = e^{i\theta}$ with possible except for $\theta = 0$ and $\theta = -\alpha$.*

Then both a number $\varepsilon_0 > 0$ and a bounded simply connected domain S_1 not containing 0 exist such that

$$\overline{S_0} \subset S_1, \quad \text{where } S_0 = \left\{ F(\zeta) : \left| \left(1 - \frac{1}{\zeta}\right)\left(1 + \frac{e^{i\alpha}}{\zeta}\right)/F'(\zeta) \right| \ge M(\alpha) - \varepsilon_0, \quad \zeta \in D^- \right\}. \tag{2}$$

Proof. First we assume that $F(|\zeta| = 1)$ is not a convex curve. Then, in view of the fact that $F(|\zeta| = 1)$ consists of two smooth arcs, on which the curvature is determined, and from simple geometric considerations, a point ζ_0, $|\zeta_0| = 1$, can be found such that

1) the curvature of $F(|\zeta| = 1)$ at $F(\zeta_0)$ is negative,
2) a smooth Jordan curve L exists connecting the points 0 and ∞ and satisfying the condition $F(|\zeta| = 1) \cap L = \{F(\zeta_0)\}$.

In the capacity of S_1 we take the annulus $\delta_0/2 < |\zeta| < 3$ with a slit along the curve L. It remains to show that $\varepsilon_0 > 0$ can be found such that (2) holds.

Assume the contrary. Then a sequence $\{\zeta_n\} \in D^-$ exists such that

$$\left| \left(1 - \frac{1}{\zeta_n}\right)\left(1 + \frac{e^{i\alpha}}{\zeta_n}\right)/F'(\zeta_n) \right| \to M(\alpha), \quad F(\zeta_n) \to A, \ \ A \notin S_1.$$

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