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A Uniqueness Theorem for Solution of One Dirichlet Problem

E. A. Utkina[1*]

[1] *Tatar State Humanitarian Pedagogical University, ul. M. Mezhlauk 1, Kazan, 420021 Russia*

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Abstract—We consider the Dirichlet problem in a four-dimensional domain formed by characteristic surfaces of an equation of the 8th order with the double major partial derivative. We state sufficient conditions for the unique solvability of this problem in terms of control coefficients, based on the method of a priori estimates.

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Introduction. The Dirichlet problem (it was studied mainly for elliptic equations) is one of main boundary value problems of mathematical physics [1, 2]. In papers [3–8] it is studied for hyperbolic equations. In this paper in the space R^4 we consider the following equation of the eighth order:

$$L(u) = D_x^{(2)} u(x) + \sum_{(\beta)<(2)} (a_{(\beta)} D_x^{(\beta)} u)(x) = f(x). \quad (1)$$

We treat it as a generalization of the two-dimensional Boussinesq–Love equation [9] which describes longitudinal waves in a thin elastic rod, taking into account transverse inertia effects. In [10] one calls equations in the form (1) pseudo-parabolic. For compactness of notation, we use multi-subscripts and multi-superscripts. For instance, if k is a positive integer number, then the symbol (k) stands for (k, k, k, k), $(\beta) = (\beta_1, \beta_2, \beta_3, \beta_4)$, $|(\beta)| = \beta_1 + \beta_2 + \beta_3 + \beta_4$, and the inequality $(\beta) < (\alpha)$ means that $\beta_i \le \alpha_i$, $|(\beta)| < |(\alpha)|$. As in [9], we treat D as the differentiation operator $D_t^k \varphi \equiv \partial^k \varphi / \partial t^k$ for $k = 1, 2, \ldots$, we do D_t^0 as the identical operator, and $D_{t_1}^{-1} \varphi \equiv \int_0^{t_1} \varphi(t)\,dt$ as the integration operator.

1. Statement of the problem. Let $D = \{0 < x_k < 1, 1 \le k \le 4\}$; denote by X_{ki} faces $\{x_k = i\}$ of the cube D $(i = 0, 1)$; assume that coefficients $a_{(i)}$ in Eq. (1) belong to the classes $C^{(i)}(\overline{D})$; $f \in C(\overline{D})$; and denote by $\{e_k\}$ an orthonormal basis in R^4 (for instance, $e_1 = (1, 0, 0, 0)$).

The problem. *Find a function* $u(x) \in C^{(2)}(D) \bigcap_{k=1}^{4} C^{e_k}(D \cup X_{k0}) \cap C(\overline{D})$ *satisfying in the domain* D *Eq.* (1) *and conditions*

$$u|_{X_{ki}} = \varphi_{ki}, \quad 1 \le k \le 4, \quad i = 0, 1. \quad (2)$$

The right-hand sides in problem (2) satisfy the natural coordination condition $\varphi_{ki} = \varphi_{rj}$ on the intersection $X_{ki} \cap X_{rj}$, $k, r = \overline{1, 4}$.

2. Solvability of the stated problem. Let us prove a uniqueness theorem for the considered problem. Clearly, it suffices to show that under homogeneous conditions (2) the homogeneous equation (1) has only the null solution. We apply the method of a priori estimates by means of the energy inequality ([2], pp. 91–96).

*E-mail: eutkina1@yandex.ru.