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Solvability of a Stationary Model of Motion of Weak Aqueous Polymer Solutions

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Abstract—In this paper we study the solvability in a weak sense of the boundary-value problem for a system of equations that describes the stationary motion of weak aqueous polymer solutions in a bounded domain with a locally Lipschitz boundary.

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The evolutionary system that corresponds to the stationary model considered in this paper has appeared in [1]; however, replacing the partial derivative in this model with the total one [2], one can treat it as a generalization of the Voigt model of fluid motion. In [3] one has experimentally confirmed that namely this system describes the behavior of weak concentrated aqueous polymer solutions (polyethylene oxide, polyacrylamide).

Papers [4–7] are devoted to the study of the strong and weak solvability of the corresponding initial boundary-value problem for the evolutionary system and its modifications. In [8] one proves the existence of a global attractor for weak solutions of the system. Naturally, the question of a full or partial description of the attractor arises. Since stationary solutions (if they exist) belong to the attractor, there naturally arises the question of the weak solvability of the boundary value problem for the corresponding stationary system

$$\sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \nu \Delta v - \varkappa \operatorname{Div}\left(v_k \frac{\partial \mathcal{E}(v)}{\partial x_k}\right) + \operatorname{grad} p = f, \quad x \in \Omega; \tag{1}$$

$$\operatorname{div} v = 0, \quad x \in \Omega; \tag{2}$$

$$v|_{\partial\Omega} = 0, \tag{3}$$

where v is the vector function of velocities at points of the domain Ω in the space $\mathbb{R}^n$, $n = 2, 3$, with the boundary $\partial\Omega$; p is the pressure function; $\mathcal{E}(v) = (\mathcal{E}_{ij})$, $\mathcal{E}_{ij} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$ is the strain rate tensor; f is the density of external forces; ν is the kinematic coefficient of viscosity, and $\varkappa$ is the delay time. The coefficient $\varkappa$ is also called *the relaxation time of deformation*.

Let Ω be a bounded domain in the Euclidean space $\mathbb{R}^n$. We denote by $\mathfrak{D}(\Omega)^n$ the space of functions defined on Ω which belong to the class C^∞, take on values in $\mathbb{R}^n$, and have a compact support in Ω. We denote by $\mathcal{V} = \{v : v \in \mathfrak{D}(\Omega)^n, \operatorname{div} v = 0\}$ the subset of solenoidal functions in the space $\mathfrak{D}(\Omega)^n$; we do by H the closure of $\mathcal{V}$ in the norm of the space $L_2(\Omega)^n$; the symbol V stands for the closure of $\mathcal{V}$ in the norm of the space $W_2^1(\Omega)^n$ with the scalar product

$$((v, w)) = \int_\Omega (\nabla v, \nabla w) dx.$$

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