

## ON THE $\aleph$ -IDENTITY OF EPIMORPHY AND EMBEDDABILITY RELATIONS ON QUASIVARIETIES

A.G. Pinus

The present article is devoted to the question of mutual conditioning of both epimorphy and embeddability relations on algebras of some varieties (quasivarieties). In the author's paper [1] the finitary independence of the epimorphy and embeddability relations was proved on algebras of any nontrivial congruence-distributive variety with extendible congruences. On the other hand, the existence of large sets of algebras of the class under consideration, on which the epimorphy and embeddability relations coincide, is of interest. In [2] it was proved that for any cardinal  $\aleph \geq \aleph_0$  in any nontrivial congruence-distributive variety  $\mathfrak{M}$  with extendible congruences a set of  $\mathfrak{M}$ -algebras possessing the cardinality  $2^\aleph$  can be found such that the epimorphy and embeddability relations between these algebras coincide; what is more, this set with these relations is isomorphic to the set of all subsets of a certain set with the cardinality  $2^\aleph$  with the relation of inclusion of the set theory. It is known that for countable  $\mathfrak{M}$ -algebras the similar result, generally speaking, is not valid. In the present article we give some sufficient conditions, under which the analogous result takes place for countable algebras of a series of varieties (quasivarieties).

Recall that if  $\mathfrak{K}$  is a certain class of universal algebras, then we denote by  $\mathcal{IK}$  the set of all types of the isomorphism of  $\mathfrak{K}$ -algebras, if  $\aleph$  is an arbitrary cardinal, then  $\mathfrak{K}_\aleph = \{\mathfrak{A} \in \mathfrak{K} \mid |\mathfrak{A}| \leq \aleph\}$ . On the set  $\mathcal{IK}$  we define two relations of quasiorder:  $\leq$  (of embeddability) and  $\ll$  (of epimorphy) as follows: for  $a, b \in \mathcal{IK}$ , the relation  $a \leq b$  ( $a \ll b$ ) takes place if and only if an algebra of type  $a$  is isomorphically embeddable into an algebra of type  $b$  (is a homomorphic image of an algebra of type  $b$ ). By a *skeleton* of epimorphy (embeddability) of class  $\mathfrak{K}$  we call the quasiordered class  $\langle \mathcal{IK}; \ll \rangle$  ( $\langle \mathcal{IK}; \leq \rangle$ , respectively), by a *countable skeleton* of epimorphy (embeddability, respectively) of class  $\mathfrak{K}$  — the quasiordered set  $\langle \mathcal{IK}_{\aleph_0}; \ll \rangle$  ( $\langle \mathcal{IK}_{\aleph_0}; \leq \rangle$ , respectively). A *double skeleton* (*double countable skeleton*, respectively) of class  $\mathfrak{K}$  is the twice quasiordered class (set, respectively)  $\langle \mathcal{IK}; \ll, \leq \rangle$  ( $\langle \mathcal{IK}_{\aleph_0}; \ll, \leq \rangle$ , respectively). More details of these concepts and related results can be found in [2], [3], and [4]. Further we shall consider only algebras of a signature which is not greater than the countable one.

Relations of epimorphy and embeddability on the class  $\mathfrak{K}$  of universal algebras are said to be *finitary (locally, see [1]) independent* if any finite twice quasiordered set is isomorphically embeddable into a double skeleton  $\langle \mathcal{IK}; \ll, \leq \rangle$  of class  $\mathfrak{K}$ . In [1] the finitary independence of the epimorphy and embeddability relations was proved on arbitrary nontrivial congruence-distributive variety with extendible congruences. More precisely, there was proved that for any variety  $\mathfrak{M}$  of this kind the twice quasiordered set  $\langle A; \leq_1, \leq_2 \rangle$  can be isomorphically embedded into  $\langle \mathcal{IM}_{\aleph_1}; \ll, \leq \rangle$ . In accordance with the results of [5] one cannot assert an analogous embeddability of arbitrary twice quasiordered set  $\langle A; \leq_1, \leq_2 \rangle$  into a countable skeleton  $\langle \mathcal{IM}_{\aleph_0}; \ll, \leq \rangle$  of an arbitrary nontrivial congruence-distributive variety with extendible congruences (even an arbitrary discriminator variety), i.e., the epimorphy and embeddability relations on countable algebras of similar varieties are not obeyed to be finitary independent.

Supported by the Russian Foundation for Basic Research (code of project 94-01-00183).

©1997 by Allerton Press, Inc.

Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of \$ 50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.