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RADIANCE OBSTRUCTIONS FOR SMOOTH MANIFOLDS OVER WEIL ALGEBRAS

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In this paper, we construct cohomology classes with coefficients in certain sheaves associated to an n-dimensional smooth manifold $M_n^{\mathbf{A}}$ over a local Weil algebra $\mathbf{A}$ which are obstructions for existence of an atlas on $M_n^{\mathbf{A}}$ whose transition functions are $\mathbf{A}$-prolongations of real diffeomorphisms. In the case of a complete manifold $M_n^{\mathbf{A}}$, these classes are trivial if and only if $M_n^{\mathbf{A}}$ is $\mathbf{A}$-diffeomorphic to the bundle of $\mathbf{A}$-velocities of some real manifold M_n.

1. Introduction

Commutative associative algebras are widely used in differential geometry starting with the papers of A.P. Kotelnikov [1], E. Study [2], W. Blaschke [3], P.A. Shirokov [4]. Under influence of papers of A.P. Norden (see, e. g., [5]–[7]), in which the algebras of complex, double, and dual numbers were used in the study of biaxial, biaffine, biplanar spaces, line geometry of non-Euclidean spaces, the geometry of smooth manifolds over algebras becomes an intensively developing direction of research. The geometry of spaces over arbitrary commutative associative algebras and their realizations was studied by A.P. Shirokov [8], [9], V.V. Vishnevskii [10], G.I. Kruchkovich [11] and other researchers (extensive lists of references can be found in [9], [10], and [12]).

A.P. Shirokov [9] discovered the structures of smooth manifolds over local algebras on tangent bundles and bundles of infinitely near points of $\mathbf{A}$-type in the sense of A. Weil [13], [14]. A smooth manifold $M_n^{\mathbf{A}}$ over a local algebra $\mathbf{A}$ ($\mathbf{A}$-smooth manifold) is called radiant if it admits an atlas agreeing with the $\mathbf{A}$-smooth structure whose coordinate changes are $\mathbf{A}$-prolongations of real diffeomorphisms, i. e., are locally of the same form as coordinate changes on Weil bundles. In [15], it was proved that a complete radiant $\mathbf{A}$-smooth manifold $M_n^{\mathbf{A}}$ is isomorphic in the category of $\mathbf{A}$-smooth manifolds to the Weil bundle $T^{\mathbf{A}}M_n$ of $\mathbf{A}$-velocities of some real manifold M_n. This result is a generalization to the case of an arbitrary local algebra of the theorem of F. Brickell and R.S. Clark ([16], theorem 5) which states that a complete nearly tangent structure on a smooth manifold is isomorphic to the standard almost tangent structure of some tangent bundle. In the case of p-tangent structures (corresponding to algebras of height 1), a similar theorem was proved by M. de León, I. Méndes and M. Salgado [17].

In this paper, we construct cohomology classes with coefficients in certain sheaves associated with $M_n^{\mathbf{A}}$ (in particular, with coefficients in the sheaf of projectable sections of the bundle $T_{\mathrm{tr}}^{\mathbf{A}}M_n^{\mathbf{A}}$ of transverse $\mathbf{A}$-velocities on $M_n^{\mathbf{A}}$ with respect to the canonical foliation) which are obstructions for $M_n^{\mathbf{A}}$ to be radiant. In the case of a complete manifold $M_n^{\mathbf{A}}$, these cohomology classes are trivial if and only if $M_n^{\mathbf{A}}$ is $\mathbf{A}$-diffeomorphic to the bundle of $\mathbf{A}$-velocities $T^{\mathbf{A}}M_n$ of some real manifold M_n. These cohomology classes can also be viewed as generalizations of the classes found by W. Goldman and M.W. Hirsch which are radiance obstructions for affine manifolds [18].