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RELATIVE σ-BOUNDEDNESS OF LINEAR OPERATORS

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Let $\mathfrak{U}$ and $\mathfrak{F}$ be Banach spaces, an operator L belong to $\mathcal{L}(\mathfrak{U}, \mathfrak{F})$, i. e., it is linear and continuous, and the operator $M : \mathrm{dom} \to \mathfrak{F}$ be linear and closed with the range $\mathrm{dom}\, M$ being dense in $\mathfrak{U}$. In following [1], we introduce into consideration an *L-resolvent* set: $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F}; \mathfrak{U})\}$ and an *L-spectrum*: $\sigma^L(M)\mathbb{C} \setminus \rho^L(M)$ of the operator M. The operator M is said to be *spectrally bounded with respect to* the operator L or (L, σ)*-bounded* (see [2], [3]) if

$$\exists a > 0\ \forall \mu \in \mathbb{C}\ (|\mu| > a) \Longrightarrow (\mu \in \rho^L(M)).$$

Let there exist an operator $L^{-1} \in \mathcal{L}(\mathfrak{F}; \mathfrak{U})$ and $\mathrm{dom}\, M = \mathfrak{U}$. Then in view of the boundedness of operator $L^{-1}M$ (or ML^{-1}), the operator M will obviously be (L, σ)-bounded. Let us analyze this notion in the case where the operator L is noninvertible, in particular, when $\ker L \neq \{0\}$.

The necessity of this analysis is as follows. As it is shown in [4], Chap. 1 (development of results can be found in [5], Chap. 1), the (L, σ)-bounded operator M plays the same role in investigation of resolvability of an equation similar to the Sobolev type equations (see [6])

$$L\dot{u} = Mu, \qquad \ker L \neq \{0\}, \tag{0.1}$$

that does the bounded operator S in the investigation of the standard equation $\dot{u} = Su$. At the present time there are known a great many of initial boundary value problems for partial differential equations, which arise in applications (see [7]), which can be reduced to the Cauchy problem $u(0) = u_0$ for equation (0.1). Moreover, the notion of a relatively σ-bounded operator is of importance in investigation of semilinear Sobolev type equations (see [8]). Finally, earlier considered problems (see [9], [10]) can be essentially simplified with use of this notion.

However, in applications, it seems very inconvenient to justify directly the (L, σ)-boundedness of operator M. Therefore, it seems to be useful sufficient conditions for relative σ-boundedness, which will relate the operators L and M to each other, without involving L-resolvent set (or L-spectrum) of the operator M.

The present article contains two parts: the first one is devoted to selection of some necessary conditions of the (L, σ)-boundedness of the operator M, and the second — to the proof of their sufficiency.

Let us agree in the following: all investigations are carried out in the real Banach spaces, but in considering "spectral" questions we introduce their intrinsic complexification; we denote by $\mathbb{I}$ and $\mathbb{O}$, respectively, the "unit" and "zero" operators, whose ranges are clear from the text.

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