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The Boundary-Value Problem for Differential-Operator Equations of the First Order in a Locally Convex Space

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Abstract—In this paper we describe a method that allows one to study boundary-value problems for the first-order differential-operator equations in an arbitrary locally convex space.

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INTRODUCTION

Many problems in Physics and Mechanics and their applications are reducible to differential-operator equations in abstract spaces. This allows one to study them by general methods and to comprehend on an abstract level multiple results on boundary-value problems stated for differential, integro-differential, and other equations.

The study of boundary-value problems in the most general form is connected with the increasing necessity to solve both special problems for well-known partial differential equations, and problems for equations different from the mentioned ones.

By now the theory of boundary-value problems for differential-operator equations is most developed in Banach (for example, [1–3]) and Hilbert (for example, [4–6]) spaces. As a rule, one studies such problems (including the questions connected with their well-posedness) with the help of the theory of semigroups [7, 8]. However, there are very few publications (at least, in Russian) dedicated to the boundary-value problems for differential-operator equations in an arbitrary locally convex space. At the same time, such problems are of great interest, because many functional spaces are nonnormable (for example, spaces of integer and analytic functions and spaces of generalized functions).

V. K. Ivanov, I. V. Mel'nikova, and A. I. Filinkov considered the following boundary-value problem ([9], P. 61):

$$u'(t) = Au(t), \;\; 0 < t < T, \;\; \mu u(0) - u(T) = u_0,$$

whose solution belongs to a Banach space E. In this paper we generalize this problem to the case of an arbitrary locally convex space. We study the obtained solution with the help of characteristics of a linear operator (namely, the order and type), as well as those of a fixed vector with respect to a linear operator introduced by V. P. Gromov [10, 11] and developed in [12–14].

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