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# ON THE WEAK LAW OF LARGE NUMBERS IN MARTINGALE TYPE $p$ BANACH SPACES UNDER A GENERAL CONDITION OF CESARO TYPE

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## Introduction

The present article is devoted to the proof of the weak law of large numbers (WLLN) of the general form

$$S_n = \sum_{j=1}^{N_n} a_{nj}(V_{nj} - c_{nj}) \xrightarrow{P} 0 \quad \text{as} \quad n \to \infty,$$

where $\{a_{nj},\ j \geq 1,\ n \geq 1\}$ is the scheme of arrays of real numbers, $\{c_{nj},\ j \geq 1,\ n \geq 1\}$ is so-called "centering" scheme of arrays, which is composed from specially selected conditional expectations, $\{N_n,\ n \geq 1\}$ is a sequence of integer random variables, and $\{V_{nj},\ j \geq 1,\ n \geq 1\}$ is a scheme of arrays of random elements definite on a probabilistic space $(\Omega, \mathcal{F}, P)$ and taking values in a separable Banach space $\mathcal{X}$ with the norm $\|\cdot\|$. A random element $S_n$ is called a *weighted sum* with *weighted coefficients*

$$\{a_{nj},\ j \geq 1,\ n \geq 1\}.$$

It is assumed that the Banach space $\mathcal{X}$ possesses the martingale type $p$, i. e., a constant $C$ exists such that for all martingales $\{S_n,\ n \geq 1\}$ with values in $\mathcal{X}$ the inequality holds

$$\sup_{n \geq 1} E\|S_n\|^p \leq C \sum_{n=1}^{\infty} E\|S_n - S_{n-1}\|^p,$$

where $S_0 \equiv 0$.

Note that any real separable Banach space has a martingale type 1, while the spaces $L_p$ and $l_p$ $(1 \leq p < \infty)$ have a martingale type $\min\{p, 2\}$. It is well-known that, if a Banach space has the martingale type $p$, then it also has the Rademacher type $p$. However, a similarity between the concepts of martingale type $p$ and Rademacher type $p$ is only superficial. A more detailed arguments can be found in [1].

The basic result of the present article is Theorem 3. It is an extension and a generalization (to the case of martingale type $p$ Banach spaces and weighted sums) of the assertions proved in [2]–[7], which were established in a scheme of arrays of random variables. Note that WLLN is proved under the assumption on the feasibility of a condition of the Cesaro type, which is more general than the condition in [4].

The question on the feasibility of WLLN under a condition of Cesaro type was first posed in [2]. Later this question was considered in [4], but with a condition of the Cesaro type replaced with a condition of the Cesaro–Hong type. Besides, both the results were established in a scheme of arrays of real random variables. An essential success in this direction was achieved in [8], where the results