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NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS WITH THE USE OF PARAMETRIC REGULARIZATION

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1. Introduction

A characteristic property of stiff systems of differential equations (SDE) is contained in the fact (see [1]–[9]) that even a small increase of integration step in used classical numerical methods (of Adams type, Runge–Kutta, etc.) can result in a drastic increase ("explosion") of error. This fact reveals a contradiction known for such SDE between a sufficiently large step of interpolation of solution obtained and an admissible step of integration. In [3]–[8] numerical methods were suggested admitting an increase of the integration step outside the boundary layer. However, the methods considered there were mainly implicit methods which are not too widely used in integration of the stiff SDE. A method in [9], which allows to eliminate the stiffness phenomenon on the basis of first integrals found beforehand (so-called truncated SDE) and, as a consequence, to ensure the required stability of the numerical methods in use, is more preferable. An a priori knowledge of an analytic expression for the first integrals of the truncated SDE is the main weakness of the mentioned method. Obviously, in practice, such an expression can be constructed only in approximate way and the arising error often causes essential distortions in the final results of the numerical integration of the stiff SDE. In [4], [5] a parametric approach to the elimination of the stiffness is developed, which is based on the idea of continuation of the solution to SDE with respect to an additional parameter, i. e., the arc length of integral curve. The main deficiency of this approach consists of a growth of computational costs if the mentioned length takes rather large values.

In this article, on the basis of the above idea concerning continuation of the solution by the parameter (not obligatory by the arc length of integral curve), we develop a general approach to the integration of the stiff SDE, which discharges a series of constraints adopted in [1]–[9] and allows us to increase the stability of numerical methods of integration.

2. Stiffness phenomenon. Statement of problem

Consider the Cauchy problem

$$\frac{dx}{dt} = f(t,x), \quad x(t_0) = x_0, \quad x \in X = X_1 \times \cdots \times X_n \in R^n, \quad t \in [t_0, T] \subset R^1, \tag{2.1}$$

where $f(t,x)$ is an n-dimensional vector function with components $f_1(t,x), \dots, f_n(t,x)$, which is continuous with respect to the time coordinate t and satisfies the Lipschitz condition in a bounded open domain $G = (t_0, T) \times X$:

$$|f_i(t, x_1', x_2', \dots, x_n') - f_i(t, x_1'', x_2'', \dots, x_n'')| \leq \\ \leq L\{|x_1' - x_1''| + |x_2' - x_2''| + \cdots + |x_n' - x_n''|\}, \quad i = 1, 2, \dots, n,$$