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MONGE HYPERSURFACES IN EUCLIDEAN SPACE

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Let us consider a Monge hypersurface M in an Euclidean space E^n, i.e., a hypersurface such that one curvature line of M is geodesic (see [1], p. 374). M is an $(n-2)$-canal surface if M is an envelope of one-parameter family of hyperspheres.

Theorem. *If a Monge hypersurface in E^n is $(n-2)$-canal, then M locally is*
1) *either a hypersurface of revolution,*
2) *or a tubular hypersurface.*

1. *Basic formulas.* Let M be a smooth hypersurface in an Euclidean space E^n. We shall use the following notation. $F(M)$ is the R-algebra of differentiable functions on M, $T^q_s(M)$ the $F(M)$-module of differentiable tensor fields of type (q,s) on M, $\chi(M)$ the Lie algebra of vector fields on M, ∂ the operator of differentiation in E_n, and $\langle\ ,\ \rangle$ the scalar product in E^n.

The Gauss–Weingarten formulas for M have the form (see [2], p. 36)

$$\partial_X Y = \nabla_X Y + \beta(X,Y)n, \quad \partial_X n = -AX,$$

where $A \in T^1_1(M)$, $X, Y \in \chi(M)$, $\beta \in T^0_2(M)$, $\beta(X,Y)$ is the second fundamental form, A is the Weingarten operator, ∇ is the Levi–Civita connection of the metric $g(X,Y) = \langle X, Y\rangle$.

In addition, the Gauss–Codazzi equations are valid:

$$R(X,Y)Z = \beta(Y,Z)AX - \beta(X,Z)AY, \qquad (1)$$
$$dA(X,Y) = 0,$$

where $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$ is the curvature tensor of ∇, $dA(X,Y) = \nabla_X AY - \nabla_Y AX - A[X,Y]$ the exterior differential of A with respect to ∇.

2. *Proof of Theorem.* Let us denote by X_i, $i = 1,\dots,n-2, U$ the unit vectors of the eigendirections of the operator A, and by k_i, $\overline{k}$ the corresponding principal curvatures of M, where the curvature line corresponding to $\overline{k}$ is a geodesic. Then we have $\nabla_U U = 0$. Taking the covariant derivative of $\langle U, U\rangle = 1$, $\langle X_i, U\rangle = 0$, we obtain

$$\langle \nabla_U X_i, U\rangle = 0, \quad \langle \nabla_{X_i} U, U\rangle = 0. \qquad (2)$$

Let us calculate

$$dA(X_i, U) = \nabla_{X_i} AU - \nabla_U AX_i - A[X_i, U] =$$
$$= (X_i\overline{k})U + k\nabla_{X_i}U - (Uk_i)X_i - k_i\nabla_U X_i - A(\nabla_{X_i}U - \nabla_U X_i) = 0. \qquad (3)$$

If we use (2) and set the coefficient at U equal to zero, then we obtain

$$X_i\overline{k} = 0, \quad i = 1,\dots,n-2. \qquad (4)$$

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