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МЕЖДУНАРОДНАЯ ФЕДЕРАЦИЯ НЕЛИНЕЙНЫХ АНАЛИТИКОВ
АКАДЕМИЯ НЕЛИНЕЙНЫХ НАУК

Проблемы Нелинейного Анализа в Инженерных Системах

Методы Подходы Гипотезы Решения ***Международный журнал***

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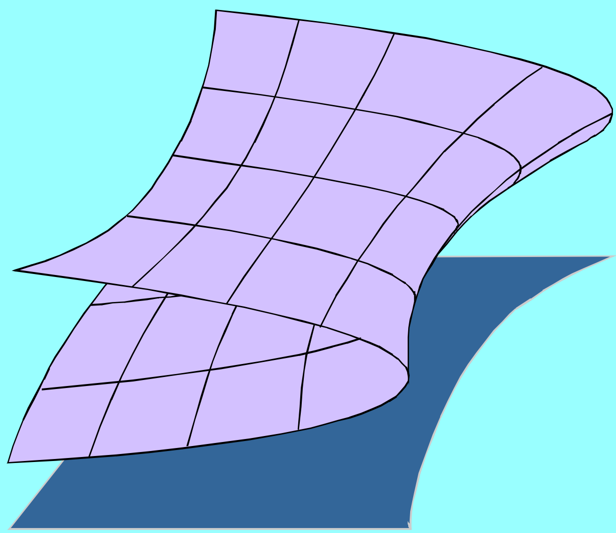

От Международного Редакционного Комитета
From International Editorial Board

Extension of the Glauber-Sudarshan mapping for Classical and quantum energy spectra

Nonlinear filters for integrated Inertial Navigation System and Global Positioning System (INS/GPS)

Критерии устойчивости трансзвуковых течений сплошной среды

Minimum relative entropy method for inverse electrocardiography problem

To dark energy theory from a Cosserat-like model of spacetime

К памяти Г.В.Каменкова, выдающегося механика, представителя Казанской Четаевской Школы механики и устойчивости

Казань

СОДЕРЖАНИЕ / CONTENTS

СОДЕРЖАНИЕ		CONTENTS
П.Вербос Расширение отображения Глаубера-Сударшана для классических и квантовых энергетических спектров	1	**P.J.Werbos** Extension of the Glauber-Sudarshan mapping for classical and quantum energy spectra
Х.Бензеррук Нелинейные фильтры для комплексированной инерциальной навигационной системы и глобальной системы позиционирования (INS/GPS)	22	**H.Benzerrouk** Nonlinear filters for integrated inertial navigation system and global positioning system (INS/GPS)
Ф.А.Слободкина Критерии устойчивости трансзвуковых течений сплошной среды	47	**F.A.Slobodkina** Stability criteria of transonic flows of continuous media
Ö.Н.Онак, Й.С.Догрусöз, Г.-В.Вебер Метод минимальной относительной энтропии для обратной задачи электрокардиографии	64	**Ö.N.Onak, Y.S.Doğrusöz, G.-W.Weber** Minimum relative entropy method for inverse electrocardiography problem
М.С.Ел Нэйши К теории темной энергии на основе модели пространства-времени Коссера	79	**M.S.El Naschie** To dark energy theory from a Cosserat-like model of spacetime
П.С.Красильников, А.Л.Куницын, С.В.Медведев К памяти Г.В.Каменкова, выдающегося механика, представителя Казанской Четаевской Школы механики и устойчивости	121	**P.S.Krasilnikov, A.L.Kunitsyn, S.V.Medvedev** To the memory of G.V.Kamenkov, outstanding mechanician, representative of Kazan Chetayev School of Mechanics and Stability
НАУЧНО-ИНФОРМАЦИОННЫЙ РАЗДЕЛ		SCIENTIFIC-INFORMATION SECTION
О.Назаренко, Е.Переверза, А.Пасичный, Д.Фишман, Г.-В.Вебер Образовательные инициативы для исследования операций *(Секция на международной конференция EURO-INFORMS 2013)*	130	**O.Nazarenko, K.Pereverza, O.Pasichnyi, D.Fishman, G.-W.Weber** Initiatives for Operational Research Education *(Section at EURO-INFORMS Conference-2013)*
Д.А.Губайдуллин Проблемы механики сплошной среды *(Научный семинар и Итоговая научная конференция 2013 года, Казань, 2013 – 2014г.)*	143	**D.A.Gubaidullin** Problems of continuum mechanics *(Scientific Seminar and Final Scientific Conference'2013, Kazan, 2013-2014)*

PROBLEMS OF NONLINEAR ANALYSIS IN ENGINEERING SYSTEMS
International Journal Kazan

Main goals of this Journal -

- to inform the specialists of appropriate fields about recent state in theory and applications; about global problems, and actual directions;
- to promote close working contacts between scientists of various Universities and Schools; between theorists and application oriented scientists;
- to mathematize the methods for solving the problems generated by engineering practice;
- to unite the efforts, to synthesize the methods in different areas of science and education.

In Journal the articles and reviews; the engineering notes; the discussional communications; the statements and solutions of problems in all areas of nonlinear analysis and their applications in engineering systems are published (including new results, methods, approaches, hypothesises,...). Authors of theoretical works should indicate the possible areas of applications in engineering practice.

The languages of publications are RUSSIAN, ENGLISH, GERMAN, FRENCH.

Edition is carried out in co-operation with Kazan Federal University (KFU), with Moscow Aviation Institute (National Research University), with International Nanobiological Testbed Ltd (INT).

INTERNATIONAL FEDERATION OF NONLINEAR ANALYSTS

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International Scientific Edition

IFNA-ANS-AAAS-RAATs - International Scientific Edition (ISE) is founded (1994) by *Kazan Chetayev School of stability and mechanics* ,under the aegis of International Federation of nonlinear analysts and Academy of nonlinear sciences.This interuniversity non-ordinary initiative raised by the intelligence of swiftly developing World brilliantly implements the objectives and goals laid in its foundations which had been formed by the following provisions:

- "True theory cannot be linear" (**A.Einstein**);
- "Unity in Diversity" (**V.Lakshmikantham**);
- **"If to be, it is necessary to be the First" (V.P.Chkalov);**
- "**Newtonian Mechanics** is an unequalled achievement of physics (natural philosophy), the whole history of human civilization. IT IS EVERLASTING. **Its powerful tree is sprouting more and more branches**. Among them there are the branches that have grown from scions grafted on this tree and cultivated in other natural sciences" (**G.G.Chyorny**);
- "Mathematics is an effective "transport" which is able to provide significant breakthrough in understanding of the essence of Environment, with deep penetration of its approaches into all the spheres including the unconventional ones".

The period of effective and successful activities resulted in establishment of ISE as a *bilingual* interdisciplinary Scientific Edition representing researches of nonlinear problems in all the diversity of basic and applied sciences. Structurally the Journal is organized as periodic Edition in two series (Journals), with preparing invited articles (as problematic character surveys) and also special topical issues on advanced scientific directions including natural sciences and the humanities (mathematics, mechanics, physics, chemistry, engineering sciences, including aviation and aerospace technologies; biological, medical, social and political sciences; ecology, cosmology, economics; nanoscience and nanotechnology, stability and sustainable development in economical, social and political systems; problems of risk and information protection, operational research, problems of higher engineering education, …).

Problems of nonlinear analysis in engineering systems (ISSN 1727–687X) http://www.kcn.ru/tat_en/science/ans/journals/ansj.html
Among the invited articles there are http://kpfu.ru/science/journals/ansj/pnaes

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– **A.G.Butkovsky**. *Some Principal Features of "Unified Geometric Theory of Control".*
– **P.J.Werbos**. *Brain-Like Intelligent Control: From Neural Nets to True Brain-Like Intelligence*
– **S.Santoli**. *Information-driven nonlinear nanoengine hierarchies for biomimetic evolware.*
– **P.J.Werbos**. *Order from chaos: a reconsideration of fundamental principles.*
– **P.Marmet.** *The overlooked phenomena in the Michelson-Morley experiment.*
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No.1(22), 2005 – *Special issue.* Integrability Problem.
No.2(23), 2005 – *Special issue.* Advances in Nanoscience and Nanotechnology.
No.3(24), 2005 – *Special issue.* Operations Research Approaches in transitional economics.
No.1(29), 2008 – *Special issue.* Advances in Nanoscience and Nanotechnology.

Actual problems of aviation and aerospace systems (ISSN 1727–6853) http://www.kcn.ru/tat_en/science/ans/journals/rasj.html
Among the invited papers there are http://kpfu.ru/science/journals/rasj/apaas

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– **K.M.Pichkhadze, A.A.Moisheev, V.V.Efanov, K.A.Zanin, Ya.G.Podobedov.** *Development of scientific-design legacy of G.N.Babakin in automatic spacecrafts made by Lavochkin Association.*
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Among the special topical issues there are

No.1(23), 2007 – *Special issue.* To the 50^{th} Anniversary of the first artificial Earth satellite launch.
No.2(24), 2007 – *Special issue.* To the 50^{th} Anniversary of the first artificial Earth satellite launch.
No.3(25), 2007 – *Special issue.* To the 50^{th} Anniversary of the beginning of Space Era.
No.1(26), 2008 – *Special issue.* To the 50^{th} Anniversary of the beginning of Space Era and the Military Space Forces Day.
No.1(32), 2011 – *Special issue.* To the 50^{th} Anniversary of the first flight of a Man in Space (Space flight of Yu.A.Gagarin).

An outstanding ability to foresee, fundamental nature, responsibility, fine qualification in the whole diversity of the problems of science, education and applications are the main constituents providing functional success of this ISE which contributes much in development of science on the whole, promotes interdisciplinary community of scientists and researchers who work in different spheres.

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From International Editorial Board

To 20-th Anniversary of International scientific Edidtion (ISE)

Problems of Nonlinear Analysis in Engineering Systems

International IFNA-ANS Journal "Problems of Nonlinear Analysis in Engineering Systems" which was established in 1994 in the depths of Kazan Aviation Institute, a member of International Scientific Associations - IFNA and ANS, had its main clear idea to unite the theorists and experimentalists, scientists and specialists of various scientific fields and concentrated on progressive and actual trends. Our Founders and Constitutors are: KAI- Tupolev KSTU (Kazan Chetayev School of Mechanics and Stability), IFNA, ANS.

KAI – Kazan Aviation Institute (Tupolev Kazan State Technical University – Tupolev Kazan National Research University). This is a well-known University which was founded on Professor N.G.Chetayev's initiative in 1932. N.G.Chetayev was a highly experienced specialist, mathematician and mechanical engineer. After returning from Germany, Gettingen University, which he had visited during his scientific trip, **N.G.Chetayev** strongly desired to create a new University which would give a fundamental engineering education in the field of aviation here in Kazan. His energetic and talented activity made KAI one of the leading Universities of aviation engineering education, and Kazan Scientific School of Mechanics and Stability, which was also founded by Academician N.G.Chetayev, graduate research and educational specialists of the highest qualification that combines fundamental science and applied problems.

IFNA – International Federation of Nonlinear Analysts is an International scientific association, which was established in 1991-1992 on initiative of professor **V.Lakshmikantam** (USA) as a transdisciplinary world society uniting specialists of different fields of knowledge including all kinds of sciences and arts from medicine to mathematics. The Association's slogan is "Unity in diversity" which is perfectly realized in reality. Nowadays IFNA includes over 300 members in different countries operating through its Departments in Russia, Ukraine, India, Bulgaria, USA, China, Italy.

ANS – Academy of Nonlinear Sciences is a Scientific Association of authorities who work actively in the field of nonlinear mathematics, physics, theory of stability, oscillations, control, decision-making and also in the area of application of nonlinear models and methods to the other natural and social sciences including biology, cybernetics, economics, ecology, aerospace and other system design, problems of security. ANS was established in 1995 on Academician **V.M.Matrosov's** (Russia) initiative and aimed to contribute to development of all scientific fields and professional consolidation of scientists who worked in different areas of theory and applications. ANS includes 4 Scientific divisions: nonlinear mathematics and applications; nonlinear mechanics and applications; nonlinear theories of stability, oscillations and control; nonlinear simulation in natural, technical sciences and liberal arts. The Academy's activity is provided by Russian and foreign members of ANS through its Regional and foreign Departments.

These joint efforts and triple alliance were conditioned by some objective circumstances, similarity of goals and objectives proposed by scientific communities, units and academic Universities, by the science and education on the whole.

The passing century's dominating trend consisting in separateness between the disciplines and deepening specialization in spheres of knowledge, is gradually changing. The gap and separation between different disciplines in science and arts steadily diminish. The ideal which can already be real consists in joining of productively and seriously working scientists from two or three absolutely different disciplines (mathematics and anthropology, political sciences and music, chemistry and philosophy, history and mathematics).

Of course, it is necessary to keep on extending academic specialization but it is also important to integrate knowledge in general, and reconstruction of perfectly entire, coordinated knowledge as the main essence of science and education is a quickly spreading idea. Moreover, the tendency to understand nonlinear world is prevailing in the majority of scientific sections. And in this the important role belongs to a **Mechanics** which jointly with Mathematics provides us with effective methodology of modelling for all spheres of knowledge.

Thus it is imminent to make a worldwide Association to strengthen the unity in diversity, to develop cooperation and interaction between nonlinear analysts from all over the world. It is the main idea of IFNA, ANS and all Founders of this scientific Journal.

"…Our Journal was established by Kazan State Technical University of A.N.Tupolev's name, International Federation of Nonlinear Analysts and Academy of nonlinear science with aim of effective issue of surveys, original, summarizing scientific articles devoted to the most essential and complex problems of nonlinear analysis…", – ***G.L.Degtyarev****, the Rector of KSTU of A.N.Tupolev's name (1995).*

"…I am pleased to address my best wishes to Kazan Aviation Institute (at present State Technical University of A.N. Tupolev's name), known for its profound traditions within the system of Higher Engineering aviation Education and currently performing as a Member of Nonlinear Analysts International Federation with additional initiatives. The issue of the International scientific Journal "Problems of nonlinear analysis in engineering systems", reflection of the most important fundamental problems and their close connection with the applied tasks will, beyond any doubt, contribute mutual enrichment in all aspects of various Schools in both scientific and educational fields; it will also enable fruitful International cooperation among Universities…", *– **A.N.Tikhonov,** the First Deputy Head of State Committee of Russian Federation on Higher Education (1995).*

"…We are sincerely pleased to commend the effective results of our interesting discussion with Academician Yu.S.Osipov (during our past meeting in Moscow) concerning joint activity within International Federation of Nonlinear Analysts. IFNA was founded as a professional scientific community under the motto "Unity in Diversity" out of the recognition of necessity to broaden the cooperation among the researchers from diverse fields of fundamental and applied sciences, for formation of the interdisciplinary unity of scientists and researchers, with the aims of progress in complex problem solving, overcoming interdisciplinary barriers.
We are also pleased to point out fruitful activity of the Russian Department of IFNA, Kazan Representation, originated in 1993 on the basis of KAI, having the initiative of the creation in Kazan the Co-ordination Center on the nonlinear problems in the engineering fields, as well as the foundation in Kazan the International Scientific Journal on general problems of the nonlinear analysis with reference to an engineering (in extended sense) systems.
It's encouraging that this University initiative, commemorating the 10-th Anniversary, generated by the spirit of contemporary intellect, has been brilliantly realized: the International bilingual Scientific Edition "Problems of nonlinear analysis in engineering systems" currently takes its noteworthy place alongside with first rate Scientific Editions, promoting the attraction of young scientific forces to the solving of Knowledge complex problems…", ***– Prof. V.Lakshmikantam,*** *President of IFNA (2004).*

"…Our joint scientific Edition is the International Journal "Problems of nonlinear analysis in engineering systems". Created by the initiative of Kazan scientists, the representatives of Kazan Chetayev's School of Stability and Mechanics, nourished with their enthusiasm and energy, conceived as a multi-languages Scientific Edition, the Journal successfully performs its function, makes considerable contribution to the development of Science on the whole, with the formation of interdisciplinary unity of scientists and researchers, working in diverse fields in Universities, Academic Spheres, Scientific Centers, industrial and state Institutions. The Journal was born at the time of clear awareness of the specific role of mathematics as an effective "transport" (pulling) means, able to provide considerable advance in the understanding of the essence of complex phenomena of the surrounding world with the deep penetration of its methods into all fields, including non-traditional for it. The time, which passed since the Journal foundation, has proved that major discoveries, essential results are obtained at the boundary of diverse scientific areas with the use of non-traditional approaches and methods.
Thus was the becoming of the International Journal "Problems of nonlinear analysis in engineering systems" as an **interdisciplinary Scientific Edition**, representing the investigations on nonlinear problems in the whole diversity of fundamental and applied sciences, including the disciplines of natural cycle and humanities (mathematics, mechanics, physics, chemistry; engineering, biological, social, political sciences; ecology, cosmology, economics, ...). It's encouraging that namely in KSTU-KAI, on the basis of which this extraordinary Journal has been issued for 10 years, there have been preserved the fundamental character, responsibility, high professional level in all the problems diversity of science, education and applications.
From all my heart I wish further success and prosperity in this activity…", ***– V.M.Matrosov,*** *Academician of RAS, President of ANS (2004).*

“…The International Scientific Journal “Problems of nonlinear analysis in the engineering systems”, initiated by the scientists of the well-known Kazan Chetayev’s School of Stability and Mechanics, founded in Kazan in 1994 on the base of A.N.Tupolev KSTU – KAI, with kindly support of the **President of Tatarstan Republic**, celebrates its 10-th Anniversary.
The main task of this extraordinary bilingual Scientific Edition is preservation and development of the heritage of mechanical and mathematical schools of Kazan and Russia on the whole, known for their deep-rooted traditions in the field of mathematics and mechanics, in the field of fundamental science and higher education.
It is noteworthy that currently this International Scientific Journal, issued under the patronage of A.N.Tupolev KSTU – KAI in cooperation with Kazan State University, is a well-known Scientific Edition with its high reputation in the scientific world. It represents research work and articles of the leading specialists on the non-linear problems in the variety of fundamental sciences, including those of interdisciplinary character (in Russian and in English, in printed and electronic versions).
It is very pleasant that the substantial heritage of **N.G.Chetayev**, an outstanding scientist, mechanicist and mathematician, the corresponding member of the Academy of Sciences of the USSR, producing great influence on science and fundamental higher education, being developed by his followers and colleagues from KSU, KAI, MSU, from Kazan and Moscow Scientific Schools of Stability and Mechanics, from the Academy of Sciences of Russian Federation and from the Academy of Sciences of Tatarstan Republic is preserved and successfully multiplied through the activity within this journal, developing the heritage of Chetayev’s School of Mechanics and Stability and extending the ideas and methods of this School to other fields of Knowledge.
This special issue, devoted to the 1000-th Anniversary of Kazan, connected with the 10-th Anniversary of the Journal, the 60-th Anniversary of Kazan Scientific Center of the Russian Academy of Sciences and other scientific events, significant for all the Partners of the Journal, proves the high status of this Scientific Edition, successfully embodying the motto “Unity in Diversity”, providing strengthening of cooperation in the world multidisciplinary community of non-linear analysts, including specialists in the field of mathematics and mechanics, physics and chemistry, biology and financial mathematics, economical, social and political sciences,…, –
M.Kh.Khasanov, *President of the Academy of Sciences of Tatarstan Republic (2005).*

Our motto for this International scientific Journal is based on important gnosiological view point of **Academician V.I.Vernadskiy,** Founder of novel theory about “**Noosphere**”:

“…We **specialize not on Sciences, but on Problems**. These Problems do not pack in frames single, determined, established domain of Science…”, – V.I.Vernadskiy.

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For discussion

Extension of the Glauber-Sudarshan mapping for classical and quantum energy spectra

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This paper begins by reviewing the general form of the Glauber-Sudarshan *P*-mapping, a cornerstone of coherence theory in quantum optics, which defines a two-way mapping between ensembles of states *S* of any classical Hamiltonian field theory and a subset of the allowed density matrices *r* in the corresponding canonical bosonic quantum field theory (QFT). It has been proven that $Tr(Hr)=E(S)$, where *E* is the classical energy and *H* is the normal form Hamiltonian. The new result of this paper is that ensembles of S of definite energy *E* map into density matrices which are mixes of eigenstates of eigenvalue zero of $N(H\text{-}E,\ H\text{-}E)$ where *N* represents the normal product. This raises interesting questions and opportunities for future research, including questions about the relation between canonical QFT and QFT in the Feynman path formulation. The Appendix gives a new result on Boltzmann states, which appear to have a link to the issue of nonclassical states and scattering experiments as discussed by Carmichael.

1. Introduction and summary

This work was motivated by two practical questions: (1) to what extent can simulations of classical partial differential equations (PDE), such as those used in nuclear phenomenology [1], approximate the correct predictions of energy spectra from the corresponding bosonic quantum field theory (QFT)?; and (2) more generally, how close can PDE simulations come to replicating the predictions of QFT?

Sections 2 and 3 of this paper will review previous work on the *P*-mapping widely used in quantum optics [2-5], and its extension to classical Hamiltonian field theory in general. For our purposes here, the key result from that work is the generalized operator trace theorem, which, when applied to the function *E*, yields:

$$Tr(H_n r(S))=E(S) \tag{1}$$

where *S* is the state of the classical system (i.e. a set of values for the fields $\underline{j}(\underline{x})$ and their duals $\underline{p}(\underline{x})$ over all points $\underline{x}$ in R^3), where H_n is the usual normal form Hamiltonian, where $E(s)$ is the classical Hamiltonian energy of the state *S*, and where $r(S)$ is the density matrix which corresponds to *S* under the *P*-mapping. From this it easily follows that:

$$Tr(H_n r) = \langle E(S)\rangle \tag{2}$$

in the case where:

$$r = \int Pr(S) r(S) d^\infty S \tag{3}$$

and where the angle brackets in (2) refer to the expectation value, for any stochastic ensemble of classical states *S*.

The operator trace theorem clearly tells us that any energy level available in ensembles of the classical system is also present in the spectrum of energy levels allowed in the corresponding QFT, *if* the energy levels are defined by the normal form Hamiltonian H_n. However, the classic text on solitons and instantons by Rajaraman [6] states that the lowest energy level available in bosonic QFT which gives rise to solitons equals the classical mass-energy *plus* positive correction terms due to stochastic effects; that would imply that the classical soliton mass-energy is lower than the quantum spectrum, and not contained within in.

[1] The views expressed here are those of the author, not those of his employer; however, as work produced on government time, it is in the "government public domain." This allows unlimited reproduction, subject to a legal requirement to keep the document together, including this footnote and authorship.

One possible way to explain this paradox is to note that different versions of QFT are being assumed here. In our work, we are relying on the canonical or Copenhagen version of QFT [7, 8], updated to reflect the widespread use of density matrices ρ rather than wave functions y in representing stochastic states, as is standard in empirical applied quantum electrodynamics (QED) [4, 5, 9]. We refer to this version as "KQFT," with "K" for Kopenhagen. (We use "K" rather than "C" because "C" for "cavity" or "circuit" in CQED is already taken, a standard term in important parts of applied QED.) By contrast, Rajaraman's discussion assumes the Feynman path version of QFT, FQFT.
Weinberg [10] reviews the history of how some branches of physics moved from reliance on KQFT to FQFT, for reasons related to the ease of proving certain renormalization results, rather than any kind of decisive empirical test. It is commonly assumed that KQFT and FQFT yield the same predictions, especially for scattering (as in [10]), but in KQFT the point of departure in making predictions is always the normal form Hamiltonian, H_n. (See section 6.3 of [8], and [11] for a nice example.) Thus in FQFT, it is generally assumed that the zero point energy or Casimir terms, which are simply deleted when defining H_n, are actually present in nature. The classic achievements of KQFT, in explaining anomalous magnetic moments and the Lamb shift, were based on the use of H_n, without the need to assume such zero-point energy terms. When zero point energy terms and other such stochastic terms are assumed, it naturally increases predicted masses, except when they are taken back away through renormalization. KQFT correctly predicts the usual flat-plate Casimir experiments on the basis of Vanderwaals forces; however, there are many other experiments which could be discussed, and the choice between KQFT and FQFT in such areas is beyond the scope of this paper.
This paper was motivated in part by the goal of finding a different explanation for the paradox. Could it be that the results for statistical ensembles might be misleading? After all, ensemble states could be mixtures of the vacuum state (zero energy) and higher energy ground states.
The main result of this paper, in section 4, is that we still have a classical-quantum equivalence for the spectra of states of definite energy, but that there is indeed a gap between classical systems and bosonic KQFT which we can quantify. More precisely, we find that any ensemble of classical states S sharing the same definite energy E can be written as:

$$r = \int_{S \in E} Pr(S)\, r\,(S)\, d^{\infty} S = \int c_a \left| y_a \right\rangle \left\langle y_a \right| d^{?} a \tag{4}$$

where the dimensionality of a depends on the specific PDE system, and where $|y_a\rangle$ are the eigenvectors of eigenvalue zero of the fundamental spectral operator:

$$M(E) = N(H_n – E,\, H_n – E) \tag{5}$$

where N refers to the normal product. In KQFT, we normally assume that the underlying states of definite energy are eigenvectors of H_n, which (because of the nonnegativity of H_n) are the same as the eigenvectors of eigenvalue zero of:

$$(H_n – E,\, H_n – E) \tag{6}$$

The difference between equation 5 and equation 6 is an operator:

$$\Delta H_n = N(H_n, H_n) – H_n H_n \tag{7}$$

To prepare for the results of section 3, section 2 will review some key pieces from the vast literature on the Glauber-Sudarshan P-mapping, in the case where the classical system to be quantized is not a field theory but a simple function of two real scalar variables p and q, like the harmonic oscillator. Glauber received the Nobel Prize for this important work in this area, which, among other things, played a key role in the development of the laser [3]. Here I will rely heavily on two sources: (1) the definitive more recent book by Carmichael [4], who provides numerous theorems, connections to empirical results, and references to earlier literature; and (2) the key paper of Mehta and Sudarshan [2] which proved the operator trace theorem. Mehta [12] provides a elegant brief recap of the history of the *P* mapping.

Section 3 briefly shows how to extend the definitions and the operator trace theorem from the case of two scalar variables *p* and *q* to the case of two real mathematical vector fields $\underline{j}(\underline{x})$ and $\underline{p}(\underline{x})$, as in Hamiltonian field theories (mathematical vector fields $\underline{j}$ can actually consist of an amalgam of relativistic covariant vector and tensor fields; thus this applies to a very general class of PDE systems). It seems likely that the results could be extended still further, to show that spatially localized classical states *S* map into density operators which are localized around the center of mass, as in the usual quantum mechanical representation of an atom with separation of coordinates; however, that is not proved here.

It should be stressed that this classical-quantum equivalence in expectation values does not imply equivalence in dynamics. As one would expect from the work on Bell's Theorem [9], we have found that the master equations which describe the classical dynamics are quite different in the general case from the usual Schrödinger equation [13].

Section 4 briefly proves that the fundamental spectral operator given in equation 5 has the property claimed. Section 5 discusses some open questions for future research related to these results.

2. The *P*-mapping for a simple (ODE) systems

2.1. General concepts

The *P* mapping provides a 1-to-1 mapping between definite states of the classical system and a subset of the density matrices for the corresponding classical system; it may be viewed either as mapping from classical states to quantum states, or as a mapping from a subset of the quantum states to classical states. Equivalently, it provides a 1-to-1 mapping between statistical ensembles of classical states and a (larger) subset of density matrices. Carmichael cites prior theorems by Sudarshan showing that any (bosonic) density matrix allowed in QFT can be represented as:

$$r = \int P(a) \left| a >< a^* \right| da \tag{8}$$

where $\left| a >< a^* \right|$ is the density matrix corresponding to the classical state a under the *P*-mapping, and where *P* is a real function; however, for some density matrices r allowed in QFT, $P(a)<0$ for some states a. Carmichael calls such density matrices "nonclassical states." Here we write the operator trace theorem (stated as equation 1.17 of [2]) as:

$$tr(rG_n) = <g(a)> \tag{9}$$

where g is some function which can be expressed as a polynomial in α and a^*, where r is the density matrix defined by equation 1, where the classical expected value (< >) is calculated based on $P(a)$ as a probability distribution, and where G_n is the operator which results from quantizing the function f as a normal product, exactly as in KQFT.

2.2. Basic equations of the P-mapping in the ODE case

Carmichael [4] shows that we can get remarkably far in understanding complex empirical phenomena like decay in two-level atoms and resonance fluorescence by applying the *P*-mapping to simple classical systems like the general harmonic oscillator:

$$H(p, q)=(p^2/2m)+(1/2)mw^2q^2 + H_I(p, q) \tag{10}$$

Carmichael's detailed discussion of the empirical details shows us that the process of an electron dropping down from one energy level to a lower energy level is far more complicated than the simplified story given in first year texts on quantum mechanics.
The classical states of this system are characterized by a complex variable which he defines on page 73, following Glauber's notation:

$$a = (mwq + ip)/(2\mathbf{h}mw)^{-1/2} \tag{11}$$

On page 75, he defines the coherent quantum states in terms of the wave function $|a>$:

$$a|a> = a\ |a> \tag{12}$$

$$<a|a^H = a^* <a| \tag{13}$$

where "a" is the usual annihilation operator and where its Hermitian conjugate, a^H, is the usual creation operator. From these definitions, he deduces (page 6) that the wave function $|a>$ obeys (and is also defined by):

$$\left|a\right\rangle = e^{-\frac{1}{2}|a|^2}\sum_{n=0}^{\infty}\frac{a^n}{\sqrt{n!}}\left|n\right\rangle \tag{14}$$

and also (on page 78) that it obeys and is defined by:

$$\left|a\right\rangle = e^{-\frac{1}{2}|a|^2}e^{a\,a^H}\left|0\right\rangle \tag{15}$$

Of course, $|a><a^*|$ is the density matrix r which the P-mapping maps the classical state a into, and it obeys equation 9.
In actuality, the P mapping also works for a broader class of systems than the Hamiltonian systems considered in this paper. Carmichael uses the P mapping to map *from* operator master equations (Schrödinger equations modified so as to approximate the effects of energy dissipating out into a reservoir) *to* sets of classical PDE – Fokker-Planck equations, which provide exact information about the level of dissipation predicted by the master equations. He also uses an extended version of equation 9, based on characteristic functions, so as to calculate two-time averages. He relies heavily on the characteristic function approach, used by Mehta and Sudarshan to establish the fact that $P(a)$ is a well-defined function for all quantum density matrices r [2]; however, when we use the mapping in the other direction, from classical ensembles to density matrices r, it is enough to focus on the case of classical ensembles for which $Pr(a)$ is a well-defined function. Mehta and Sudarshan use the same relationships, using "z" to represent what we (following Glauber) will call "a".
Given any polynomial or analytic function of two variables, p and q, Mehta and Sudarshan [2] note that the function may be represented equivalently as a polynomial function of a and a^*, where $a=p+iq$. And so, if we write:

$$g(a, a^*) = \sum_{k,j} A_j^k a^j (a^*)^k \tag{16}$$

Substituting from equations 12 and 13, equation 16 implies:

$$g(a, a^*)\left|a\right\rangle\left\langle a\right| = \sum_{k,j} A_j^k a^j \left|a\right\rangle\left\langle a\right|(a^H)^k \tag{17}$$

If we quantize g by quantizing α as the operator a and a^* as a^H, the raw quantized version of $g(a, a^*)$ would be:

$$G_r = \sum_{k,j} A_j^k a^j (a^H)^k \tag{18}$$

From equation 17, for $|a\rangle\langle a|$ of trace 1 (a consequence of equations 14 and 15), and using a well-known trace identity, we can deduce:

$$g(a,a^*) = Tr(g(a,a^*)|a\rangle\langle a|) = Tr(\sum_{k,j} A_j^k (a^H)^k a^j |a\rangle\langle a| \tag{19}$$

Equation 9 then falls out directly by using a basic property of normal products of expressions such as equations 16 and 18:

$$G_n = \sum_{k,j} A_j^k (a^H)^k a^j \tag{20}$$

3. Extended *P*-mapping to general Hamiltonian field theories

3.1. Review of the extension of the *P* mapping to Hamiltonian field theories

Here we consider the more general situation where we replace p and q by the real mathematical vector fields $\underline{j}(\underline{x})$ and $\underline{p}(\underline{x})$ defined over $\underline{x}$ in R^3. A state S of this system is simply a set of values for $\underline{j}(\underline{x})$ and $\underline{p}(\underline{x})$ across all $\underline{x}$ in R^3. In place of the Hamiltonian shown in equation 9, we assume (as in [13]) a classical Hamiltonian of the form:

$$\boldsymbol{H} = H = \int \left(\frac{1}{2} \sum_{j=1}^{n} \left(|\nabla j_j|^2 + m_j^2 j_j^2 \right) + f(\underline{j}, \underline{p}, \nabla \underline{j}) \right) d^3 \underline{x} \tag{21}$$

Mehta and Sudarshan [2] refer to prior work establishing similar relationships for multiple modes or state variables, a_j, and for continuous state variables, which would include state variables indexed by the momentum coordinate $\underline{p}$ in R^3. In our case, we replace equation 15 by:

$$|S\rangle = Z e^{\sum_j \int a_j(\underline{p}) a_j^H(\underline{p}) d^3 \underline{p}} |0\rangle \tag{22}$$

where Z is the real scalar which normalizes $|S\rangle$ to length 1. Because $a_k(\underline{p}')$ commutes with all the terms in the exponent of equation 14, except for the one where $j=k$ and $\underline{p}'=\underline{p}$, it is easy to see that equation 12 generalizes to:

$$a_k(\underline{p})\,|S\rangle = a_k(\underline{p})\,|S\rangle \tag{23}$$

and to its Hermitian conjugate:

$$\langle S|\, a_k^H(\underline{p}) = a_k^*(\underline{p})\,\langle S| \tag{24}$$

The Hamiltonian H in equation 21 is a fairly complicated polynomial in the momentum representation, in the set of state variables $j_j(p)$ and $p_j(\underline{p})$ across all $\underline{p}$, involving integrals over $\underline{p}$ (due to Fourier convolution) and appearance of $\underline{p}$ itself (due to the gradient term); however, it is still a polynomial, like equation 16, but with more terms. The logic of equations 16 through 20 still goes through, giving us the generalized operator trace theorem:

$$Tr(rG_n) = \langle g(S) \rangle \tag{25}$$

Before applying equation 25 to physical systems, we need to decide how to map the state variables $a_j(\underline{p})$ into physical variables of the system, just as Carmichael did for the systems he considered. By analogy to equation 11, we have proposed [15\3]:

$$a_j(\underline{p}) = q_j(\underline{p}) + i t_j(\underline{p}) \tag{26}$$

where:

$$q_j(\underline{p}) = \sqrt{w_j(\underline{p})}\int e^{-i\underline{p}\cdot\underline{y}} j_j(\underline{y}) d^d \underline{y} \tag{27}$$

$$t_j(\underline{p}) = \frac{1}{\sqrt{w_j(\underline{p})}}\int e^{-i\underline{p}\cdot\underline{y}} p_j(\underline{y}) d^d \underline{y} \tag{28}$$

$$w_j(\underline{p}) = \sqrt{m_j^2 + |\underline{p}|^2} \tag{29}$$

The normal form quantization of H (or P) results from substituting $F_j(\underline{x})$ and $P_j(\underline{x})$ for $j_j(\underline{x})$ and $p_j(\underline{x})$ in equation 21, and mapping classical multiplication into normal forms, where F_j and P^j are defined precisely as in chapter 7 of Weinberg[10]:

$$F_j(\underline{x}) = F_j^+(\underline{x}) + F_j^-(\underline{x}) \tag{30}$$

$$P_j(\underline{x}) = P_j^+(\underline{x}) + P_j^-(\underline{x}) \tag{31}$$

where:

$$F_j^-(\underline{x}) = \left(F_j^+(\underline{x})\right)^H \tag{32}$$

$$P_j^-(\underline{x}) = \left(P_j^+(\underline{x})\right)^H \tag{33}$$

$$F_j^+(\underline{x}) = c\int \frac{e^{i\underline{p}\cdot\underline{x}} a_j(\underline{p})}{\sqrt{w_j(\underline{p})}} d^d \underline{p} \tag{34}$$

$$P_j^+(\underline{x}) = -ic\int \left(\sqrt{w_j(\underline{p})}\right) e^{i\underline{p}\cdot\underline{x}} a_j(\underline{p}) d^d \underline{p} \tag{35}$$

Inserting these definitions into equation 25, and taking normal products, we arrive at the special case of equation 25 most relevant to our purposes here:

$$Tr(rH_n) = \langle H(S) \rangle \tag{36}$$

where H_n is the normal form Hamiltonian quantized with the bosonic field operators given in Weinberg [10], and $H(S)$ is the classical Hamiltonian. Of course, the same goes through for the momentum operator and for functions of energy and momentum

4. The fundamental spectral operator for *P*-mapped classical ensembles

The main result of this paper was already stated in the introduction. The claim is that for any ensembles of states of S of a Hamiltonian field theory, all of which have the same definite classical energy level E, the density matrix r (under the P-mapping) is spanned by eigenvectors of eigenvalue zero of the fundamental spectral operator $M(E)$ given in equation 5. This result follows very directly from the previous results given above. Choose the function:

$$g(S) = (H(S) - E)^2 \tag{37}$$

where we now use the function H to represent the classical energy function (classical Hamiltonian), and E is a nonnegative real number. Clearly the energy level equals E for all states in the ensemble if and only if $g(S)$ is zero for all states in the ensemble; by the nonnegativity (and lack of multiple zeroes) of g, this is true if and only if the expected value of g in the ensemble is zero. By equation 25, this is true if and only if $G_n = M(E)$ obeys:

$$Tr(M(E)r) = 0 \tag{38}$$

But note that $M(E)$ is a Hermitian operator, nonnegative over all density matrices r. Thus it has an orthogonal eigenvector/eigenspace decomposition, with all eigenvalues zero or positive. Likewise, r it, being Hermitian and positive, has its own decomposition into rank 1

states corresponding to its eigenvectors. Because all of the terms in the resulting expansion of $Tr(M(E)r)$ are zero or positive, equation 38 can only be satisfied if all of the eigenvectors of r are orthogonal to all of the eigenspaces of $M(E)$, except for the eigenspace of eigenvalue zero (any component of r in the other eigenspaces would yield a positive term, invalidating equation 38). Thus r is made up of rank one terms, all made of vectors in that eigenspace of $M(E)$.
Note that we could have chosen any other polynomial or analytic function g, which has the property that $g(S) = 0$ for states of energy E and >0 for states of other energy. $G(S)^2$, for example, has the same property. This would yield other operators with the same basic property as $M(E)$, but more complicated. Also, in applying this concept, one may easily extend it to sets of S restricted to zero total momentum or to some desired gauge or rotation angle, if applicable.
Also note that we cannot construct in general a new version of the Hamiltonian operator by simply patching together the zero-eigenvalue eigenspaces of $M(E)$ for different values of E, because these eigenspaces are not in general orthogonal to each other.

5. Questions for the future

Several questions emerge from these results.
The first question is: how could we best apply these new mathematical connections in practical areas, such as quantum optics, simulation modeling, or predicting the emergent properties of classical nonlinear dynamical systems? That is an important question, but it gets into many large and complex areas, beyond the scope of this paper.
The second question is – could there be implications for the formulation of QFT itself? Given that the underlying difference between KQFT and FQFT seems to involve the role of the normal form Hamiltonian H versus the raw Hamiltonian, is it possible that predicting spectra based on $M(E)$ rather than equation 6 could be consistent with empirical reality?
At first, this seems unlikely, but equation 7 is actually very close to part of what we actually do in KQFT; see sections 6.3 and 7.1 of Mandl and Shaw [8]. It is basically just the usual second order contraction, which is what we use to calculate the self-energy of the electron. Section 9.6.1 of Mandl and Shaw reminds us that the first great success of QED was in predicting the anomalous magnetic moment of the electron, by Schwinger in 1948. The correction which he applied was in fact based on the second-order contraction term, rather than any use of time-independent perturbation theory to revise the estimated eigenvalue. For higher-order corrections, Schwinger has noted [14, 15] that we can get consistent and accurate predictions simply by bootstrapping the use of physical mass and second-order connections. For other, more routine calculations of atomic and molecular spectra in applied QED, the self-energy corrections are small compared to what is used in applications, but would presumably be similar. Even for more complicated systems in applied QED, such as predicting energy levels in semiconductors, one of the most successful methods has been the Non-equilibrium Green’s Function (NGEF) method, which grew out of Schwinger’s approach to self-consistent propagators. In fact, it would be interesting to see how much of all this could be deduced as an exercise in phenomenological modeling, similar to Schwinger’s source theory, but with density matrices rather than wave functions as the basis of the bootstrapping.
To extend this kind of spectral modeling to the analysis of unexplained spectral data in the nuclear sector [16] could be very important, but would require discussion of which Lagrangian to use, as well as the literature on bosonization, which is far beyond the scope of this paper (to get a feeling for the size of the bosonization literature, one may go to Google Scholar, and branch forward from the list of papers which cite [11], one of the original seminal papers in that field). It should be noted that if a Lagrangian is chosen which contains topological Higgs terms, it becomes necessary to map the fields into a kind of equivalent

vacuum-dependent representation before the *P*-mapping, like the j_0 subtraction used in electroweak theory today, to ensure L2 integrability. Of course, because of the classical-quantum equivalence, this approach always results in finite mass-energies. In a similar vein, Schwinger noted the finite nature of his self-energy correction methods [1415]. When PDE simulation is used (as in [1]), it is not necessary to have convergent Taylor series or perturbation expressions in order to calculate key spectral predictions.

Appendix: Thermodynamic properties of classical Hamiltonian systems

This paper has presented a test for an ensemble of classical states *S* to be an ensemble of definite (uniform) energy. For a large class of classical nonlinear dynamical systems, ODE or PDE, it is also possible to characterize the invariant equilibrium ensembles in a similar manner. More precisely, we show how to extend the Boltzmann distribution (and the more general class of equilibrium ensembles of which it is an example) to the class of Hamiltonian systems which we call "statistically incompressible".

For the ODE case, consider the usual Fokker-Planck equation, using "$p(\underline{x})$" to represent the density of probability at point $\underline{x}$ in state space:

$$\dot{p} + (\underline{v} \cdot \nabla p) + p(div\underline{v}) = 0 \tag{39}$$

Here, the state space is just the space of possible values for the two vectors $\underline{j}$ and $\underline{p}$ in R^n. From equation 39, the uniform measure $d^n\underline{j}\,d^n\underline{p}$ (whose gradient is zero) will be an invariant measure (have the property that p dot will be zero) if:

$$"div\underline{v}" = \sum_{i=1}^{n} \frac{\partial \dot{j}_i}{\partial j_i} + \sum_{i=1}^{n} \frac{\partial \dot{p}_i}{\partial p_i} \tag{40}$$

We will call a Hamiltonian ODE system "statistically incompressible" if its dynamics have this property. Likewise, a Hamiltonian PDE system will be called statistically incompressible whenever the measure $d^\infty S = d^{n\infty}\underline{j}(\underline{x})d^{n\infty}\underline{p}(\underline{x})$ is invariant under the dynamics of the PDE.

Given any function $g(E, \underline{I})$, where E is energy and $\underline{I}$ is the set of other conserved quantities of the system, it is obvious that $g(E, \underline{I})\ d^\infty S$ will also be an invariant measure if the system is statistically incompressible. If this measure meets the requirements for a probability distribution (that it be nonnegative and that its integral equals one), then it represents an equilibrium (ergodic) distribution of states *S*. The generalized Boltzmann distribution is a function of this form ($g = c\ \exp(-kE-\underline{c}\cdot\underline{I})$).

In addition to the usual primal representation *r* of any ensemble of states *S*, as given in equation 3, there is also a dual representation *F* defined by:

$$\begin{aligned} Pr(S) &= p(S)d^\infty S \\ Tr(F\,r(S)) &= p(S) \end{aligned} \tag{41}$$

Thus the operator version of the Boltzmann distribution is not the usual primal version of the Boltzmann operator, but the dual representation defined by:

$$F = G_n(E,\underline{I}) \tag{42}$$

where *g* is the usual Boltzmann function (an analytic function).

A previous paper [17] showed how the Bell's Theorem experiment could be predicted by either of two local realistic models – using the discrete mathematics of Markov Random Fields, which provide a kind of equilibrium statistical analysis across space and time, and are allowed under a loophole in the theorem. That paper concluded with the question of how to generalize that type of analysis from the discrete case to the case of continuous variables and fields. It is hoped that the observations in this appendix will be of some help in clarifying and answering that question. To make that connection, it is also important to note that dual operators like equation 42 remain valid, even when the density of allowed states is constrained by boundary conditions, as in the classic Planck black body analysis. It is possible to construct a Boltzmann approach to the probability of the continuous range of possible trajectories through an experiment, such that Carmichael's quantum trajectory approach [18] gives a good approximation to what happens when there are only two high-probability paths for a photon entering a polarizer at other than its preferred polarization angle, from either the input or output side.

The primal and dual *P* mapping concepts can be extended to stochastic ODE and PDE as well, at the cost of some complexity, beyond the scope of this paper; however, stochastic factors at the initial, terminal, and intermediate "reservoir" levels appear sufficient to track this type of experiment. CQED effects can be modeled as a consequence of the boundary condition of the matter which ultimately absorbs a photon, in the future, without a need to assume zero-point energy in the intervening space.

The immediate motivation of this work is to address modeling issues for applied QED, as discussed in [17]. Yet some may be concerned that the development of coherence theory mathematics applicable to simulations of other field theories, such as the work in [1], might be risky in some ways. Manton [1] argues in chapter 11 that the application of coherence theory is the key missing element needed to enable some very ambitious goals being pursued in Russia for nuclear technology. However, I would argue that this is not a realistic concern until and unless more realistic PDE models (Lagrangian) are available, and that is a major challenge in and of itself. Considering the larger picture of national and global security, the need for progress in applied QED and other future technologies with substantial barriers to entry outweighs any second-order risk which might exist, in my view. Tradeoffs between information restriction and national security are discussed in detail at nss.org/itar.

The fundamental goal of this work is to begin to carry out step one of the three step program for return to reality in physics, given in section 2.5 of [19].

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Nonlinear filters for integrated Inertial Navigation System and Global Positioning System (INS/GPS)

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The research of the Sigma-Point Kalman filters (SPKF), Unscented Kalman filter (UKF), Central Difference Kalman filter (CDKF), and divided difference filters (DDF) application in aerospace domain becomes more and more extended, all these algorithms are generally applied and compared to the most popular tool in estimation which is an Extended Kalman Filter (EKF). In this paper, the first order divided difference filter and the second order divided difference filter have been applied to integrated navigation system including inertial measurement unit (IMU) and global positioning system (GPS). These filters have been tested and applied to low cost inertial sensors with very low accuracy. Results are discussed and compared under different conditions [1-19].

1. Introduction

In low cost integrated system, the Kalman filter [1, 6] is generally used to combine the outputs of IMU, linear accelerations and angular rate for strap down configuration with kinematical model of vehicle. Global Position System (GPS) outputs such as position and velocity are used to correct Inertial Navigation System (INS) errors growing in time. This correction is possible using Kalman filter in the linear model case and extended Kalman filter in the nonlinear case, which is the most useful filter for integrated navigation system problems [6, 13, 17]. The main problem in inertial navigation system is the bias and drift of the accelerometers and gyroscopes. In fact the problem of filtering is followed by control problem resolution in order to improve and realize more accurate control algorithm based on more accurate estimates [2, 3, 8, 9]. To solve this problem different approaches can be used such as indirect and direct mode and different filters can be applied. The first approach means estimation of the state vector errors using linear Kalman filter [6] and summing these values at the output of the inertial system design. This is why it is called an indirect mode. For the second approach, i.e. the direct mode, the state vector is estimated directly via nonlinear estimator like EKF or other nonlinear filter like Sigma-Points Kalman filters [13, 15, 18, 19]. So, the mean values and covariance of the state vector can be estimated better than by EKF, because for these kind of estimators the accuracy equals to the second and the third order of Taylor development, and nonlinear equations are directly used to propagate the Sigma-Points through the state system equation and the measurement equation. The UKF uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points. For the CDKF, it adopts an alternative method in linearization called a central difference approximation, like the UKF. Some other new algorithms were introduced in [7]. They are divided difference filters with polynomial Stirling's interpolation [11], interpolation of the first and the second order are employed and tested. They are called Divided difference 1^{st} order filter and Divided Difference 2^{nd} order filter. To compare and test the efficiency of these new algorithms, we use very low accuracy inertial sensors with a very high bias and drift. They are compared and tested in terms of position, velocity and attitude estimation for a high speed aerial vehicle.

2. Inertial Navigation System

The inertial navigation system is based on inertial sensors like accelerometers and gyroscopes for gyro-stabilized inertial navigation systems, but usually these are very expensive. Strap down inertial navigation systems are employed, using gyrometers. For better understanding of inertial navigation let us distinguish the different frames included in this kind of navigation as in the figure 1 below.

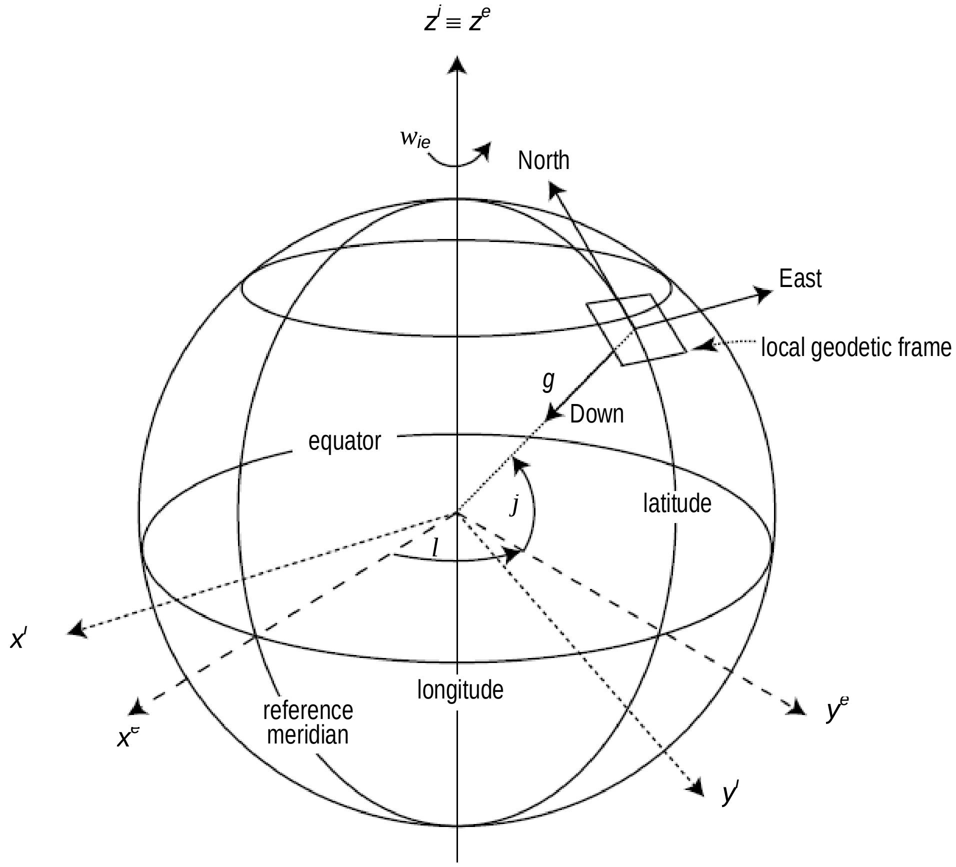

Fig.1.

It is marked: inertial frame (i), Earth frame (e) and navigation frame (n)

The inertial frames have to be considered in order to develop the inertial integration equations. Let us describe the different equations used in inertial navigation.

2.1. Position and attitude integration

The mechanization of strap down inertial navigation is done as in [4] and [10]. The attitude of the vehicle like yaw, pitch and roll angles are obtained using the following integration [14, 6]:

$$\dot{\alpha} = \begin{pmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{pmatrix} = C_{\dot{\Phi}\dot{\Theta}\dot{\Psi}/pqr} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \frac{1}{\cos\Theta} \begin{pmatrix} \cos\Theta & \sin\Phi\sin\Theta & \cos\Phi\sin\Theta \\ 0 & \cos\Phi\cos\Theta & -\sin\Phi\cos\Theta \\ 0 & \sin\Phi & \cos\Phi \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \qquad (1)$$

p, *q*, *r*: angular velocities in body axes.
According to the direction cosines matrix and attitude integration matrix:

$$R_b^n = \begin{pmatrix} \cos\Theta\cos\Psi & -\cos\Phi\sin\Psi + \sin\Phi\sin\Theta\cos\Psi & \sin\Phi\sin\Psi + \cos\Phi\sin\Theta\cos\Psi \\ \cos\Theta\sin\Psi & \cos\Phi\cos\Psi + \sin\Phi\cos\Theta\sin\Psi & \sin\Phi\cos\Psi + \cos\Phi\cos\Theta\sin\Psi \\ -\sin\Theta & \sin\Phi\cos\Theta & \cos\Phi\cos\Theta \end{pmatrix} \qquad (2)$$

and
$$C_{\dot{\Phi}\dot{\Theta}\dot{\Psi}/pqr} = \begin{bmatrix} 1 & \sin\Phi\tan\Theta & \cos\Phi\tan\Theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi\sec\Theta & \cos\Phi\sec\Theta \end{bmatrix}$$

it is calculated:
$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}_{measured} - R_b^n \begin{bmatrix} \Omega \cos f + \dot{l} \cos f \\ \dot{f} \\ -\Omega \sin f - \dot{l} \sin f \end{bmatrix} \tag{3}$$

Let h : altitude; f: latitude; l: longitude; Ω : Earth angular rate; R_M: meridian Rayon of the Earth; R_T: tangential Rayon of the Earth.
For position and velocity integration the following equations are used in North, East and down directions in the navigation frame (N). Indices “M: meridian” and “T: tangential”.

$$v^n = \begin{pmatrix} v_N \\ v_E \\ v_B \end{pmatrix} = \begin{pmatrix} (R_M + h) & 0 & 0 \\ 0 & (R_T + h)\cos f & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{f} \\ \dot{l} \\ \dot{h} \end{pmatrix} \tag{4}$$

Where $r^{LLa} = [j \quad l \quad h]^T$ is the vector of latitude, longitude and altitude of the vehicle.
It is then possible to integrate the last equation to obtain the position in the navigation frame using the following equation:

$$\dot{r}^{LLa} = \begin{pmatrix} \dot{j} \\ \dot{l} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} (R_M + h) & 0 & 0 \\ 0 & (R_T + h)\cos f & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} v_N \\ v_E \\ v_B \end{pmatrix} = Dv^n \tag{5}$$

It is easy to understand more the inertial mechanization by observing the diagram given in fig.2.

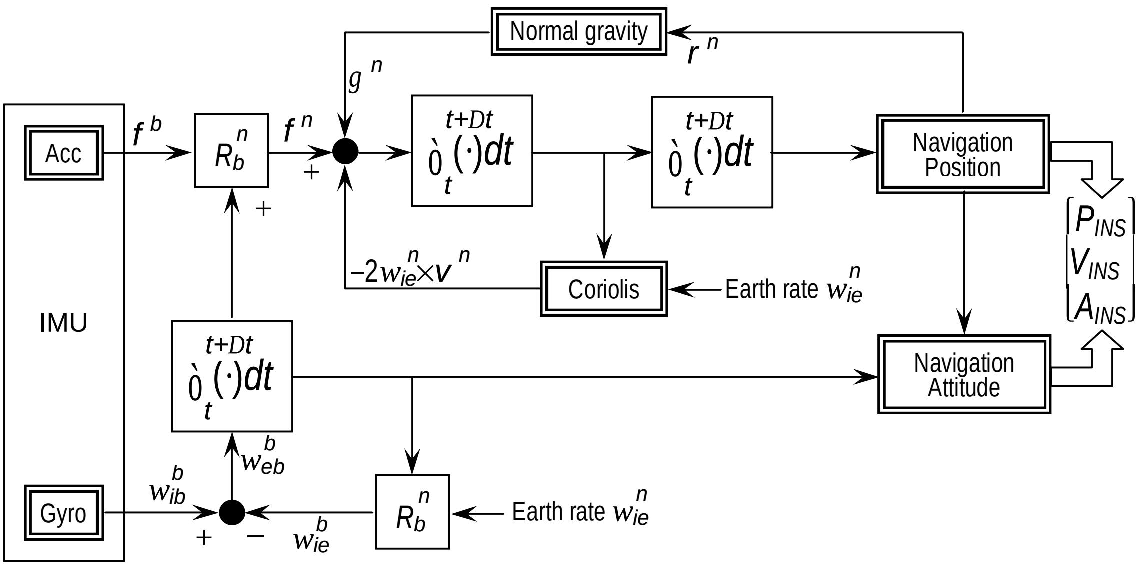

Fig.2. Strap down Inertial Navigation Integration

$$\begin{aligned} a_{output} &= a_{input} + d\,a_{bias} + K_a a_{input} + K_0 + K_1 a_{input} + K_2 a_{input}^2 + \dots \\ w_{output} &= w_{input} + dw_{bias} + K_w w_{input} + L_0 + L_1 w_{input} + L_2 w_{input}^2 + \dots \end{aligned} \tag{6}$$

All parameters and coefficients are defined in [6, 10, 12, 13]. Inertial navigation system presents some advantages and disadvantages given below:
- Advantages: complete output solution, good accuracy during short time, high data rate and small size.

- Disadvantages: accuracy decrease after a long time of navigation, gravity sensitivity and obligatory external aid for initialization process (exp: GPS).

2.2 Errors in inertial sensors

The main errors in the inertial system are: bias, scale factors and nonlinearity as in the following equation:

3. External aid for inertial navigation

To correct the inertial navigation system, an external aid such as radio navigation system is used, providing solutions in position and velocity vectors [6]. In the best case, attitude of the vehicle is possible to estimate in special case with multiple GPS receiver, such as presented in figure 2 [13, 6]. In this paper, it is assumed that the external aid is the global positioning system outputs with position, velocity and using three receivers of attitude angles of the vehicle. P – position, V – velocity, A – attitude are integrated as given below.

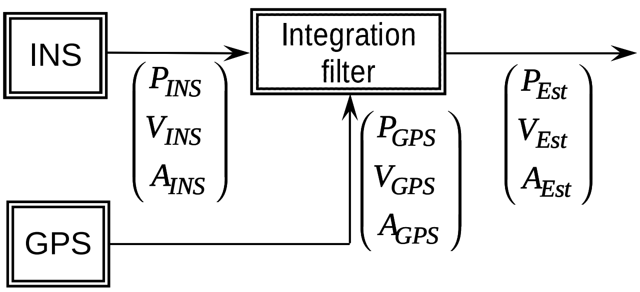

Fig.3. Integrated navigation system using GPS as external aid of navigation.

4. State equations

In this case, it is useful to integrate positions using north, east and down distances without latitude, longitude and altitude as presented in previous paragraphs. Velocities are integrated in north, east and down directions and the angles integration provide yaw, pitch and roll angles of the vehicle. State equations in discrete time could be written like in [12]:

$$\begin{bmatrix} p_n(k) \\ v_n(k) \\ y_n(k) \end{bmatrix} = \begin{bmatrix} p_n(k-1)+v_n(k-1)\Delta t \\ v_n(k-1)+\left\{C_n^b(k-1)\left[f_b(k)+d\,f_b(k)+g^n\right]\right\}\Delta t \\ y_n(k-1)+E_b^n(k-1)\left[w^b(k)+dw^b(k)\right]\Delta t \end{bmatrix} + \begin{bmatrix} w_{p^n}(k) \\ w_{v^n}(k) \\ w_{y^n}(k) \end{bmatrix} \tag{7}$$

where $x(k) = f(x(k-1), u(k), w(k))$ is the state vector. It estimates positions, velocities and attitude angles of vehicles [12].

$$E[w_v(k)] = 0$$

$$E[w_v(k)w_v(k)^T] = Q(k) = \begin{bmatrix} s_{f^b}^2 & 0 \\ 0 & s_{w^b}^2 \end{bmatrix} \tag{8}$$

Q – system covariance noise matrix.

Where

$$\nabla f_x(k) = \begin{bmatrix} \frac{\partial p^n(k)}{\partial p^n(k-1)} & \frac{\partial p^n(k)}{\partial v^n(k-1)} & \frac{\partial p^n(k)}{\partial y^n(k-1)} \\ \frac{\partial v^n(k)}{\partial p^n(k-1)} & \frac{\partial v^n(k)}{\partial v^n(k-1)} & \frac{\partial v^n(k)}{\partial y^n(k-1)} \\ \frac{\partial y^n(k)}{\partial p^n(k-1)} & \frac{\partial y^n(k)}{\partial v^n(k-1)} & \frac{\partial y^n(k)}{\partial y^n(k-1)} \end{bmatrix} \tag{9}$$

The observation equation from GPS is linear such as given below:

$$Z_{k+1} = H(X_k) + V_k \tag{10}$$

Where observation matrix is as follows:

$$H_k = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{11}$$

$$E[V_k(k)] = 0$$

with
$$E[V_k(k)V_k(k)^T] = R(k) \tag{12}$$

$R(k)$ is the covariance noise matrix of GPS measurement. The noise is assumed to be white Gaussian additive noise. In practice, this measurement covariance is well estimated and does not represent an obstacle in estimation problem, which is not the case for System covariance.

5. Filtering

In this section five algorithms are presented; extended Kalman filter, Sigma-Point Kalman filters (SPKF) and divided difference filters (DDF). Filters are described as in the following subsections.

5.1 Extended Kalman filter (EKF)

It is the most frequently used technique in nonlinear filtering. Each time the calculation algorithm is used, the nonlinear dynamic and measurement functions are approximated to the first order of Taylor development around the current estimates.
The algorithm of EKF is given below [5, 13]:

Initialisation: $\hat{X}_0$ et P_0.

Prediction:

$$\hat{X}_{k+1/k} = f_k(\hat{X}_k) \tag{13}$$

$$P_{k/k-1} = F_k(\hat{X}_k)P_{k-1}F_k(\hat{X}_k)^T + Q_k \tag{14}$$

Filtering:

$$K_k = P_{k/k-1}\, H_k{}^T(\hat{X}_{k/k-1})\left[H_k(\hat{X}_{k/k-1})P_{k/k-1}H_k{}^T(\hat{X}_{k/k-1}) + R_k\right]^{-1} \tag{15}$$

$$\hat{X}_k = \hat{X}_{k/k-1} + K_k\left[Z_k - h_k(\hat{X}_{k/k-1})\right] \tag{16}$$

$$P_k = P_{k/k-1} - K_k\, H_k(\hat{X}_{k/k-1})P_{k/k-1} \tag{17}$$

$$k=k+1 \tag{18}$$

Extended Kalman filter is the most useful filter in all engineering domains and especially in aerospace applications.

5.2. Sigma-Point Kalman filters

Sigma-point Kalman filters use deterministic sampling points to capture the mean and the covariance of the estimate; according to the definitions [4] it is possible to present both UKF and CDKF (resp.) as given in [19]:

Initialisation: $\hat{X}_0$ et P_0.
For k=1…….∞;
t=k-1

$$\hat{x}_0 = E[x_0], \quad P_{x_0} = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] \tag{19}$$

$$\hat{x}_0^a = E\left[x_0^a\right] = \left[\hat{x}_0^T \bar{v}_0^T \bar{n}_0^T\right]^T$$

$$P_0^a = E\left[\left(x_0^a - \hat{x}_0^a\right)\left(x_0^a - \hat{x}_0^a\right)^T\right] = \begin{bmatrix} P_{x_0} & 0 & 0 \\ 0 & R_v & 0 \\ 0 & 0 & R_n \end{bmatrix} \tag{20}$$

sigma-points calculation

$$c_t^a = \left[\hat{x}_t^a \;\; \hat{x}_t^a + g\sqrt{P_t^a} \quad \hat{x}_t^a - g\sqrt{P_t^a}\right] \tag{21}$$

sigma-point propagation $c_{k/t}^x = f\left(c_t^x, c_t^v, u_t\right)$ (22)

$$\hat{x}_k^- = \sum_{i=0}^{2L} w_i^m c_{i,k/t}^x \tag{23}$$

$$P_{x_k}^- = \sum_{i=0}^{2L} w_i^c \left(c_{i,k/t}^x - \hat{x}_k^-\right)\left(c_{i,k/t}^x - \hat{x}_k^-\right)^T \tag{24}$$

filtering

$$Y_{k/t} = h\left(c_{k/t}^x, c_t^n\right) \tag{25}$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} w_i^m Y_{i,k/t} \tag{26}$$

$$P_{\tilde{y}_k} = \sum_{i=0}^{2L} w_i^c \left(Y_{i,k/t} - \hat{y}_k^-\right)\left(Y_{i,k/t} - \hat{y}_k^-\right)^T \tag{27}$$

$$P_{x_k y_k} = \sum_{i=0}^{2L} w_i^c \left(c_{i,k/t}^x - \hat{x}_k^-\right)\left(Y_{i,k/t} - \hat{y}_k^-\right)^T \tag{28}$$

update

$$K_k = P_{x_k y_k} P_{\tilde{y}_k}^{-1} \tag{29}$$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(y_k - \hat{y}_k^-\right) \tag{30}$$

$$P_{x_k} = P_{x_k}^- - K_k P_{\tilde{y}_k} K_k^T \tag{31}$$

where

$$g = \sqrt{L + l}, \quad w_0^m = \frac{l}{L + l} \tag{32}$$

$$w_0^c = w_0^m + \left(1 - a^2 + b\right), \; w_i^c = w_i^m = \frac{1}{2(L + l)} \tag{33}$$

and

$$1e - 3 < a < 1, \quad b = 2, \quad k = 0 \tag{34}$$

For UKF, the propagation is done in one step and propagated through the nonlinear function of the dynamic and measurement [5]. It is possible to compare it with CDKF algorithm such as given in the following sections [13, 18, 19].

5.3. Central Difference Filter (CDKF)

For the CDKF it is approximately the same idea as in UKF algorithm. When propagation steps of Sigma-Points through the nonlinear functions of the system dynamics and measurement equation are different, the nonlinear approximation of these functions is done using the divided differences [13].

Initialisation

$$\hat{x}_0 = E[x_0], \quad P_{x_0} = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] \tag{35}$$

For k=1…∞

t=k–1

Sigma-Points calculation

$$\hat{x}_t^{av} = [\hat{x}_t \;\; \bar{v}] \tag{36}$$

$$P_t^{av} = \begin{bmatrix} P_{x_{k-1}} & 0 \\ 0 & R_v \end{bmatrix} \tag{37}$$

$$c_t^{av} = \left[\hat{x}_t^{av}, \quad \hat{x}_t^{av} + h\sqrt{P_t^{av}}, \quad \hat{x}_t^{av} - h\sqrt{P_t^{av}} \right] \tag{38}$$

Sigma-Point propagation

$$c_{k/t}^{x} = f\left(c_t^{x}, c_t^{v}, u_t\right) \tag{39}$$

$$\hat{x}_k^{-} = \sum_{i=0}^{2L} w_i^m c_{i,k/t}^{x} \tag{40}$$

$$P_{xk}^{-} = \sum_{i=1}^{L}\left[w_i^{c1}\left(c_{i,k/t}^{x} - c_{i+L,k/t}^{x}\right)^2 + w_i^{c2}\left(c_{i,k/t}^{x} + c_{i+L,k/t}^{x} - 2c_{0,k/t}^{x}\right)^2 \right] \tag{41}$$

measurement Sigma-Points calculation

$$\hat{x}_{k/t}^{an} = \left[\hat{x}_k^{-} \;\; \bar{n}\right] \tag{42}$$

$$P_{k/t}^{an} = \begin{bmatrix} P_{x_k}^{-} & 0 \\ 0 & R_n \end{bmatrix} \tag{43}$$

$$c_{k/t}^{an} = [\hat{x}_{k/t}^{an}, \quad \hat{x}_{k/t}^{an} + h\sqrt{P_{k/t}^{an}}, \quad \hat{x}_{k/t}^{an} - h\sqrt{P_{k/t}^{an}}] \tag{44}$$

update

$$Y_{k/t} = h\left(c_{k/t}^{x}, c_{k/t}^{n}\right) \tag{45}$$

$$\hat{y}_k^{-} = \sum_{i=0}^{2L} w_i^m Y_{i,k/t} \tag{46}$$

$$P_{y_k} = \sum_{i=1}^{L}\left[w_i^{c1}\left(Y_{i,k/t} - Y_{i+L,k/t}^{-}\right)^2 + w_i^{c2}\left(Y_{i,k/t} + Y_{i+L,k/t}^{-} - 2Y_{0,k/t}\right)^2 \right] \tag{47}$$

$$P_{y_k x_k} = \sqrt{\left(w_1^{c1} P_x^{-}\right)}\left[Y_{1:L,k/t} - Y_{L+1:2L,k/t}\right]^T \tag{48}$$

$$K_k = P_{x_k y_k} P_{\tilde{y}_k}^{-1} \tag{49}$$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(y_k - \hat{y}_k^-\right) \tag{50}$$

$$P_{x_k} = P_{x_k}^- - K_k P_{\tilde{y}_k} K_k^T \tag{51}$$

with

$$h = \sqrt{3}\,,\ w_0^m = \left(h^2 - L\right)/h^2\,,\ w_i^m = \frac{1}{2h^2}\,,\ w_i^{c1} = \frac{1}{4h^2}\,,\ w_i^{c2} = \left(h^2 - 1\right)/4h^4 \tag{52}$$

Generally, it is advisable to use the optimal value of h=1 [7] and the filter is called then 2nd order Divided difference filter DD2.

5.4. 1st order divided difference filter DD1

As a starting point of the derivation of the 1st order divided difference filter, the basic structure of the Kalman filter can be assumed [7]. Other interpolation techniques can be developed and used [16].

1st Order Divided Differences Filter

$$S_k = chol(P_k^{'}) \tag{53}$$

$$\hat{z}_k^{'} = h_k\left(\hat{x}_k^{'}\right) \tag{54}$$

$$P_{z,k}^{'} = H\left(\hat{x}_k^{'}, S_k, h\right) H^T\left(\hat{x}_k^{'}, S_k, h\right) + R_k \tag{55}$$

$$P_{x,z}^{'} = S_k H^T\left(\hat{x}_k^{'}, S_k, h\right) \tag{56}$$

$$K_k = P_{x,z}^{'}\left(P_{z,k}^{'}\right)^{-1} \tag{57}$$

$$\hat{x}_k = \hat{x}_k^{'} + K_k\left(z_k - \hat{z}_k^{'}\right) \tag{58}$$

$$P_k = P_k^{'} - K_k P_{z,k}^{'} K_k^T \tag{59}$$

$$S_k = chol\left(P_k\right) \tag{60}$$

$$\hat{x}_{k+1}^{'} = f_k\left(\hat{x}_k\right) \tag{61}$$

$$P_{k+1}^{'} = F\left(\hat{x}_k^{'}, S_k, h\right) F^T\left(\hat{x}_k^{'}, S_k, h\right) + Q_k \tag{62}$$

with:

$$H\left(\hat{x}_k^{'}, S_k, h\right) = \left\{H_{j,i}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{h_{j,k}\left(\hat{x}_k^{'} + h * s_{x,i}\right) - h_{j,k}\left(\hat{x}_k^{'} - h * s_{x,i}\right)\right\}/2*h \tag{63}$$

$$F\left(\hat{x}_k^{'}, S_k, h\right) = \left\{F_{j,i}\left(\hat{x}_k^{'}, S_k, h\right)\right\} = \left\{f_{j,k}\left(\hat{x}_k^{'} + h * s_{x,i}\right) - f_{j,k}\left(\hat{x}_k^{'} - h * s_{x,i}\right)\right\}/2*h \tag{64}$$

S^k is the Cholsky decomposition of the covariance matrix P^k. The functions f_k and h_k are approximated using a Stirling polynomial interpolation at the first order of nonlinear system equation.

5.5. 2nd order Divided difference filter DD2

As presented in the previous section, the algorithm uses the second order polynomial Stirling interpolation, moreover two other matrices are to be defined comparing with the 1st order divided difference filter DD1. These matrices are presented in the following expressions:

$$H^{(2)}\left(\hat{x}_k^{'},S_k,h\right)=\left\{H_{j,i}^{(2)}\left(\hat{x}_k^{'},S_k,h\right)\right\}=\left\{\frac{\sqrt{h^2-1}\left(h_{j,k}\left(\hat{x}_k^{'}+hs_{x,i}\right)+h_{j,k}\left(\hat{x}_k^{'}-hs_{x,i}\right)-2h_{j,k}\left(\hat{x}_k^{'}\right)\right)}{2h^2}\right\} \quad (65)$$

$$F^{(2)}\left(\hat{x}_k^{'},S_k,h\right)=\left\{F_{j,i}^{(2)}\left(\hat{x}_k^{'},S_k,h\right)\right\}=\left\{\frac{\sqrt{h^2-1}\left(f_{j,k}\left(\hat{x}_k^{'}+hs_{x,i}\right)+f_{j,k}\left(\hat{x}_k^{'}-hs_{x,i}\right)-2f_{j,k}\left(\hat{x}_k^{'}\right)\right)}{2h^2}\right\} \quad (66)$$

$$H\left(\hat{x}_k^{'},S_k,h\right)=\left\{H_{j,i}\left(\hat{x}_k^{'},S_k,h\right)\right\}=\left\{h_{j,k}\left(\hat{x}_k^{'}+h*s_{x,i}\right)-h_{j,k}\left(\hat{x}_k^{'}-h*s_{x,i}\right)\right\}/2*h \quad (67)$$

$$F\left(\hat{x}_k^{'},S_k,h\right)=\left\{F_{j,i}\left(\hat{x}_k^{'},S_k,h\right)\right\}=\left\{f_{j,k}\left(\hat{x}_k^{'}+h*s_{x,i}\right)-f_{j,k}\left(\hat{x}_k^{'}-h*s_{x,i}\right)\right\}/2*h \quad (68)$$

F and *H* are called Auxiliary matrix; $F^{(2)}$ and $H^{(2)}$ are called second order auxiliary matrices.
<u>2nd Order Divided Differences Filter.</u>
2nd Order filter can be written by the following equations, as given in [7][11]:

$$S_k=chol\left(P_k^{'}\right) \quad (69)$$

$$\hat{z}_k^{'}=\frac{h^2-n_x}{h^2}h_k\left(\hat{x}_k^{'}\right)+\frac{1}{2h^2}\sum_{i=1}^{n_x}\left(h_k\left(\hat{x}_k^{'}+hs_{x,i}\right)+h_k\left(\hat{x}_k^{'}-hs_{x,i}\right)\right) \quad (70)$$

$$P_{z,k}^{'}=H\left(\hat{x}_k^{'},S_k,h\right)H^T\left(\hat{x}_k^{'},S_k,h\right)+H^{(2)}\left(\hat{x}_k^{'},S_k,h\right)H^{(2)T}\left(\hat{x}_k^{'},S_k,h\right)+R_k \quad (71)$$

$$P_{xz,k}^{'}=S_kH^T\left(\hat{x}_k^{'},S_k,h\right) \quad (72)$$

$$K_k=P_{xz,k}^{'}\left(P_{z,k}^{'}\right)^{-1} \quad (73)$$

$$\hat{x}_k=\hat{x}_k^{'}+K_k\left(z_k-\hat{z}_k^{'}\right) \quad (74)$$

$$P_k=P_k^{'}-K_kP_{z,k}^{'}K_k^T \quad (75)$$

$$S_k=chol\left(P_k\right) \quad (76)$$

$$\hat{x}_{k+1}^{'}=\frac{h^2-n_x}{h^2}h_k\left(\hat{x}_k^{'}\right)+\frac{1}{2h^2}\sum_{i=1}^{n_x}\left(f_k\left(\hat{x}_k^{'}+hs_{x,i}\right)+f_k\left(\hat{x}_k^{'}-hs_{x,i}\right)\right) \quad (77)$$

$$P_{k+1}^{'}=F\left(\hat{x}_k,S_k,h\right)F^T\left(\hat{x},S_k,h\right)+F^{(2)}\left(\hat{x}_k^{'},S_k,h\right)F^{(2)T}\left(F^{(2)}\left(\hat{x}_k^{'},S_k,h\right)\right)+Q_k \quad (78)$$

After describing the different algorithms used in this paper, let us present below the different simulations applied to INS/GPS integration using EKF, UKF, CDKF, DD1and DD2, with and without GPS outliers.

6. Simulation

In our work the duration of simulation is 25 seconds in the first part, then augmented to 100s in the second part of simulation. EKF, UKF and CDKF are implemented using MATLAB software, with and without selective Availability of GPS outputs conditions are assumed, and it is assumed under white Gaussian environment. At first, EKF and SPKF are compared. Under selective availability conditions, these filters were applied using the following data:
Integration time interval Δt =0.005s, receiver noise=10m, accelerometer bias=(0.05…1)g, gyrometer bias=(0.02…2)°/s, velocity=(150…220)m/s, initial uncertainty in North direction: 1000m, initial uncertainty in East distance: 1000m, initial uncertainty in Down distance: 100m, initial uncertainty in VN: 5m/s, initial uncertainty in VE: 5m/s, initial uncertainty in

VD: 10m/s, initial uncertainty in j (yaw): 1°, initial uncertainty in θ (pitch): 1°, initial uncertainty in y (roll): 1°, and GPS data in SA conditions are augmented from 10m to 1000 m during 40 seconds for positions, from 5m/s to 50m/s for velocities and from1° to 10° for attitudes angles. Concerning the initialization step of the three filters, it was 80% from the true values of the state vector. GPS data output frequency equal to 10Hz and the inertial integration process was preceded at frequency equal to 200 Hz. The figures, illustrating these results, are do not shown here; some comments are given below.

The second simulation was carried out in order to show the efficiency of DD1 and DD2 comparing to EKF which is a reference filter in this paper. Sample time Δt=0.005s, receiver noise=10m, accelerometer bias=1m/s, gyrometer bias=2°/s, velocity=(150…220)m/s, initial Uncertainty in North direction: 1000m, initial Uncertainty in East direction: 1000m, initial Uncertainty initial in Down direction: 100m, initial Uncertainty in VN: 5m/s, initial Uncertainty in VE: 5m/s, initial Uncertainty in VD: 10m/s, initial Uncertainty in φ(yaw): 1°, initial Uncertainty in θ (pitch): 1° , initial Uncertainty in ψ (roll): 1°, and GPS data in SA conditions are augmented from 10m to 1000 m during 40 seconds for positions, from 5 m/s to 50m/s for velocities and from 1° to 10° for attitudes angles. Concerning the initialization step of the three filters, it was 80% from the true values of the state vector. GPS data are used at the frequency equal to 10Hz and the inertial integration process was at frequency equal to 200 Hz. Some comments are below.

These simulations were made in order to prove the efficiency of the divided difference filters comparing to the extended Kalman filter. The analysis shows that EKF cannot track the true trajectories or the true values of the state vector elements. The simulations were repeated under selective availability conditions. Interesting results are obtained.

So, when the selective availability is introduced, it is possible to observe that DD1 and DD2 return immediately to the tracked trajectory, on the contrary the EKF takes more time to track and return to the desired values. For the attitude estimation the same observations can be made, in addition to the fact that DD1 and DD2 present some instability in the attitude angles during selective availability period.

7. Conclusion

Having tested different algorithms to estimate the linear and nonlinear models, it is possible to conclude that all these algorithms work well and give good results without real difficulties with the exception of UKF for which three parameters are to be set properly. It is therefore worth noting that it would be preferable to implement CDKF rather than UKF since it shows the competing performance without these parameters. But execution time is also important. SPKF offers better solutions for velocity than EKF in general conditions, but when SA is introduced, time is required to return to the true values of the estimated state vector. In this case SPKF with this model do not surpass EKF significantly because the nonlinearity is only present in the dynamic state equation, which is used only for the prediction step. It is also possible to conclude that DD1 and DD2 provide better results than EKF and SPKF in all simulation conditions due to the low accuracy of the inertial sensors. EKF cannot estimate the true positions, velocities and attitude accurately in contrast to interpolation filters that are centred to the true trajectories and provide high efficiency. Under selective availability conditions, when DD1 and DD2 return immediately after the end of disturbance period, the EKF takes more time to track the true values. To obtain the equivalent results using EKF, it is necessary to estimate and augment the state vector with bias and drift estimation. However, DD1 and DD2 without augmentation give very accurate results comparing to EKF. It can be also noted that DD1 and DD2 provide the same estimation accuracy because in this paper the nonlinear function in the implemented model is only present in the system equation. However, the measurement equation itself is linear. Some instability are observed using the interpolation filters in angles estimation due to the nonlinear model of the three angles (yaw, pitch and roll) that were integrated using Euler modelling. So, in the future it will be interesting to modify the attitude integration model, to obtain more stable estimation and confirm these results by real-life experiments with digital signal processing and by applying these algorithms to a nonlinear model both in system and measurement equations.

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Stability criteria of transonic flows of continuous media

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Research of stability of steady gas-dynamic flows corresponding to the research of stability of steady solutions of hyperbolic systems of equations is an important theoretical and practical task [1, 2]. An ability to predict the behavior of a disturbed solution allows early selection of the domain of governing parameters, which provides stable operation of the developed device.
Research of stability assumes the existence of a known steady solution $U(x)$, which is subjected to small disturbances $u^*(x, t)$. Mathematical description of time evolution of small disturbances answers the question of steady solution stability.

1. Small disturbances in the vicinity of critical point of steady flow. The case of equations with continuous right-hand sides

This section deals with the behavior of unsteady disturbances in the vicinity of the critical point of steady solutions of quasilinear hyperbolic systems of differential equations.
A point is called critical, if one of characteristic velocities of the system vanishes at it. In gas dynamics and physical gas dynamics it means that the flow reaches sound velocity. Generalizing the terminology to the solutions of arbitrary quasilinear hyperbolic systems of differential equations, we will call solutions with one zero crossing characteristic velocities transonic ones.
Consider a hyperbolic system of quasilinear equations for n functions $u_j(x, t)$ in a characteristic form

$$l_j^i(u_k,x)\left[\frac{\partial u_j}{\partial t}+c^i(u_k,x)\frac{\partial u_j}{\partial x}\right]=f^i(u_k,x),\quad i,j,k=1,...,n. \tag{1}$$

We will assume that the components of matrix (l_j^i), characteristic velocities c^i and right-hand sides of equations $f^i(u_k, x)$ are continuous and differentiable functions of all arguments u_k, x.
The case of piecewise continuous functions $f^i(u_k, x)$ will be considered later. Let one of the characteristic velocities of the system of equations become zero (e.g. $c^1(u_k, x)$) in the considered range of variables u_k, x, and the rest of characteristic velocities $c^m(u_k, x) \neq 0$ (m = 2, ..., n).
Let us choose a steady solution $U_j(x)$ (j = 1, 2, ..., n), which intersects the surface $c^1(U_k, x) = 0$ at some point x^* and is continuous in its small vicinity. The point of intersection $U_j(x)$ (j = 1, 2, ..., n) with the surface $c^1(u_k, x) = 0$ will be called a critical one and we will take it as an origin of a spatial coordinate x and U_j (j =1, 2, ..., n) value, so we have c^1 (0, 0, ... , 0) = 0 for steady solutions at the critical point.
For steady solutions the system (1) transforms into a system of ordinary differential equations.

$$l_j^i(U_k,x)c^i(U_k,x)\frac{dU_j}{dx}=f^i(U_k,x) \tag{2}$$

Since the characteristic velocity $c^1 = 0$, the first row of matrix of coefficients at dU_j/dx becomes zero, and at the points of surface $c^1 = 0$ the matrix rank is $n - 1$. At the rest of the points of the considered domain this rank is n.
If the function f^1 is continuous and is not zero at the points of surface $c^1 = 0$, then derivatives dU_j/dx go into infinity at these points and change sign when crossing $c^1 = 0$. This means that continuous and unambiguous over x solution exists in a one-sided neighborhood of a critical point, and such points may be considered as one of the boundaries of the class within which the solution is examined

If the critical point is an inner point of a class, then existence of continuous and unambiguous over x solution is provided by the sign change of function $f^{1}(U_k, x)$ at a critical point.
Points of space, in which conditions

$$c^{1}\left(U_k, x\right) = 0, \quad f^{1}(U_k, x) = 0 \tag{3}$$

are simultaneously met, are singular points of steady equations (2.2).
Assume that the steady solution is weakly disturbed, i.e. solution of equation (1) is a sum

$$u_j(x,t) = U_j(x) + u_j^*(x,t)$$

of steady $U_j(x)$ and small unsteady disturbance

$$u_j^*(x,t)$$

Consider the behavior of solution $u_j(x,t)$ of the system (1) in the small vicinity (with size d) of the critical point $x = 0$, at which $c^1 = 0$, $f_1 = 0$ and $U_j(x)=0$. Since it is supposed that $U_j(0)=0$, then u_j^* coincides with u_j at the critical point due to the assumption of weak unsteady disturbance.
Let us denote a limit value of l_j^i at $x = 0$, $u_k = 0$ by $l_{j_0}^i$ and introduce new variables – Riemann invariants

$$w_i(x,t) = l_{j_o}^i u_j; \quad u_j = r_{j_k} w_k; \quad r_{j_k} = (l_{j_0}^i)^{-1} \tag{4}$$

Since the matrix $l_{j_0}^i$ is nonsingular, the matrix r_{j_k} is nonsingular, too.
Owing to the fact that the solution of equation (1) is considered within small vicinity of a critical point, the values $u_j(x,t)$ are small, and consequently $w_j(x,t)$ are small, too.
Assume that w_j are of the order of d , and x is of the order of d.
Let us expand coefficients and right-hand sides of equation (1) in a series over w_k and x, keeping dominant terms in the notation. Denote $w_1 \equiv w$, and the rest of values mark with Greek index m: w_m, where $m = 2, ..., n$.

$$l_{j_0}^1 \left[\frac{\partial u_j}{\partial t} + \left(c_1^1 w + c_m^1 w_m + c_x^1 x\right) \frac{\partial u_j}{\partial x} \right] = f_1^1 w + f_m^1 w_m + f_x^1 x \tag{5}$$

$$l_{j_0}^m \left[\frac{\partial u_j}{\partial t} + c_0^m \frac{\partial u_j}{\partial x} \right] = f_0^m, \quad (i, j, k=1, \dots, n, \quad m = 2, \dots, n); \quad l_{j_0}^i = l_j^i(0,0) \tag{5*}$$

$$c_1^1 = \frac{\partial c^1}{\partial w}(0,0); \quad c_m^1 \frac{\partial c^1}{\partial w_m}(0,0); \quad c_x^1 = \frac{\partial c^1}{\partial x}(0,0); \quad f_0^m = f^m(0,0); \quad f_1^1 = \frac{\partial f^1}{\partial w}(0,0);$$

$$f_m^1 = \frac{\partial f^1}{\partial w_m}(0,0); \quad f_x^1 = \frac{\partial f^1}{\partial x}(0,0).$$

Notation (0,0) shows that the value is calculated at the critical point, where $x = 0$, $U_j = 0$.
System (5*) may be written in invariants

$$\frac{\partial w_m}{\partial t} + c_0^m \frac{\partial w_m}{\partial x} = f_0^m$$

Keeping the terms of the order of unity in the equations, one can find quasisteady solution neglecting the term $\frac{\partial w_m}{\partial t}$, which is of the order of *d*. Integrating we obtain

$$w_m = a_m x + w_{m_0}(t); \quad a_m = \frac{f_0^m}{c_0^m}$$

Using this result for the transformation of equation (3), keeping the terms of the order of *d* in the equation, we will obtain

$$\frac{\partial w}{\partial t} + \left[c_1^1 w + \left(c_x^1 + c_m^1 a_m\right)x + c_m^1 w_{m_0}(t)\right]\frac{\partial w}{\partial x} = = f_1^1 w + \left(f_x^1 + f_m^1 a_m\right)x + f_m^1 w_{m_0}(t) \qquad (6)$$

Summation over m index is assumed. Let us reduce the obtained equation to a simpler form introducing new variable

$$c = c_1^1 w + \left(c_x^1 + c_m^1 a_m\right)x \qquad (7)$$

Multiplying (5) by c_1^1, we obtain

$$\frac{\partial c}{\partial t} + \left(c + j\,(t)\right)\frac{\partial c}{\partial x} = a\,c + b\,x + y\,(t) \qquad (8)$$

$$a \;= f_1^1 + c_1^1 + c_m^1 a_m$$

$$b \;= c_1^1\left(f_x^1 + f_m^1 a_m\right) - f_1^1\left(c_x^1 + c_m^1 a_m\right)$$

$$j\;(t) = c_m^1 w_{m_0}(t)$$

$$y\,(t) = c_1^1 f_m^1 w_{m_0}(t) + \left(c_1^1 + c_m^1 a_m\right) j\,(t)$$

Note that in many cases functions $w_{m_0}(t)$ may be considered equal to zero, since these values are determined by disturbances getting into the vicinity of the critical point and associated with characteristic velocities c_m. ≠ 0.

Anyway, assuming that $j\;(t)$ and $j_t{}'(t)$ are small, one can rename the variables including $j\;(t)$ and $y\;(t)$ in them, which reduces equation (8) to the same form as when

$$j\,(t) \;=\; y\,(t) = 0$$

$$\frac{\partial c}{\partial t} + c\frac{\partial c}{\partial x} = a\,c + b\,x \qquad (9)$$

Equation (9) is solved integrating the equations of characteristics

$$\frac{dc}{dt} = a\;\;c + b\;\;x, \qquad \frac{dx}{dt} = c \qquad (10)$$

Note that $c = u - a \approx a_0(M - 1)$, $M = u/a$ – Mach number, a_0 – sound velocity at the critical point of steady solution) in gas dynamics and in many problems of physical gas dynamics.

Equation (9) describes both unsteady and steady solutions in the vicinity of the critical point $x = 0$.
In the steady case, system (10) yields the solution $c(x)$ of equation (9) in a parametric form

$$c = c(t), \quad x = x(t).$$

2. Qualitative analysis of the system of equations $\frac{dc}{dt} = a c + b x, \quad \frac{dx}{dt} = c$

To provide the existence of continuous solutions of system (10) that pass through the critical point, it is necessary for the characteristic equation of this system, determining the eigenvalues

$$l^2 - a\ l\ - b\ = 0$$

to have real roots l_1, l_2. To be definite, let us assume $l_1 > l_2$.
Behavior of integral curves in x, c plane for different real values l_1, l_2 is shown in fig.1; 2; 3, where arrows correspond to growing t.
Eigendirections corresponding to l_1 and l_2 at a singular point are described by equations

$$c = l_1 x, \quad c = l_2 x.$$

Any steady solution $c(x)$ consisting of unambiguous over x segments of integral curves passing through a singular point can be considered as an undisturbed solution.
Consider an arbitrary domain in x, c plane. The domain has an area of S and is bounded by a closed curve, the points of which move according to equations (10).
Since the velocity field described by the right-hand sides of (10) has a constant divergence

$$\frac{1}{S}\frac{dS}{dt} = \frac{¶}{¶x}\left(\frac{dx}{dt}\right) + \frac{¶}{¶c}\left(\frac{dc}{dt}\right) = a\ = l_1 + l_2$$

the change of area S in time is described by equation $S = S_0 \exp a t$, where S_0 is an area at $t = 0$.
The area $\int dcdx$ of an arbitrary disturbance $dc(x,t) = c(x,t) - c(x)$ bounded in space, where dc is measured from an arbitrary integral curve of system (10) chosen as an undisturbed solution (fig.4), changes in the same manner
If the disturbance dc is not concentrated within some arbitrary chosen segment $[x_1, x_2]$, then considering the change of area $\int_{x_1}^{x_2} dcdx$ over this segment, it is necessary to allow for the area flux through lines $x = x_1$, $x = x_2$, so that

$$\frac{d}{dt}\int_{x_1}^{x_2} d\,cdx = a\int_{x_1}^{x_2} d\,cdx + \int_{c(x_1)}^{c(x_1)+dc(x_1)} cdc - \int_{c(x_2)}^{c(x_2)+dc(x_2)} cdc =$$

$$= a\int_{x_1}^{x_2} d\,cdx + \frac{1}{2}\left[2c(x_1) + d\,c(x_1)\right]d\,c(x_1) - \frac{1}{2}\left[2-(x_2) + d\,c(x_2)\right]d\,c(x_2) \qquad (11)$$

Consider an unsteady solution and two close points the coordinates of which differ by Δx and the values of Δc differ by Δc (fig.4). Since the system (10) is linear, then Δc and Δx also

satisfy this system. Fig.1, 2, 3 show that if the initial value of ratio $\Delta c/\Delta x > l_2$, then this value tends to the limit value l_1 in time (l_1 is a tangent of the inclination angle of an eigenvector).
Derivative *dc/dx* tends to a limit value like $\exp[-(l_1 - l_2)t]$, and a characteristic of undisturbed flow passing through a singular point tends to a critical point like $\exp l_2 t$ at $l_2 < 0 < l_1$ or like $\exp l_1 t$ at $l_2 < l_1 < 0$. Hence the limit inclination of the derivative is reached sooner than an undisturbed characteristic approaches the critical point in the case of $l_2 < 0 < l_1$ and later in the case of $l_2 < l_1 < 0$.
In the last case continuous equation no longer exists, and at the subsequent moment of time the solution with discontinuities should be considered. Thus, a disturbance of finite length over *x* tends to acquire triangular or saw-tooth shape.
It is well-known that in weak shock waves the increments of all values with the accuracy up to the second-order terms of increments coincide with the increments of values in the corresponding simple wave. Thus, the existence of weak shock wave does not lead to the emergence of disturbances of other variables $w_2,\ldots,w_n$ behind it. Shock wave velocity with the same accuracy is equal to half-sum of characteristic velocities calculated using the states before and after the shock wave. Hence the existence of weak shock wave will not lead to additional, as compared to continuous case, time change of $\int d\,c dx$ integral, and equation (11) remains valid.
Although hyperbolic systems are considered, the result, however, remains valid for more general systems, since here only one assumption is adopted (characteristic corresponding to the shock wave is not multiple).
Let us consider the behavior of solutions of equation (9) with different combinations of l_1 and l_2 signs. Growth of disturbances $dc=c(x, t)-c_0(x)$ in time means instability of steady solution $u_{k0}(x)$, while attenuation of disturbances does not means stability of $u_{k0}(x)$ over a finite segment of *x* axis, since the reason for instability may be uncoupled with the behavior of solution in the vicinity of the critical point. However, we will briefly refer to the last case as to stable solution.

1. In the case $l_1 > l_2 > 0$, a singular point is a node with positive eigendirections. Behavior of integral curves of system (10) for this singularity is demonstrated in fig.1.
It is evident from equation (10) and fig.1 that any continuous disturbance different from zero at $x = 0$ at $t = 0$ grows in time without limit in the vicinity of this point. Leading and falling edges of such disturbance will move away from the critical point. If one of the edges is a shock wave, then its velocity away from the critical point grows with the intensity of shock wave.
Thus, any steady solution passing through the critical point of the considered type is unstable.

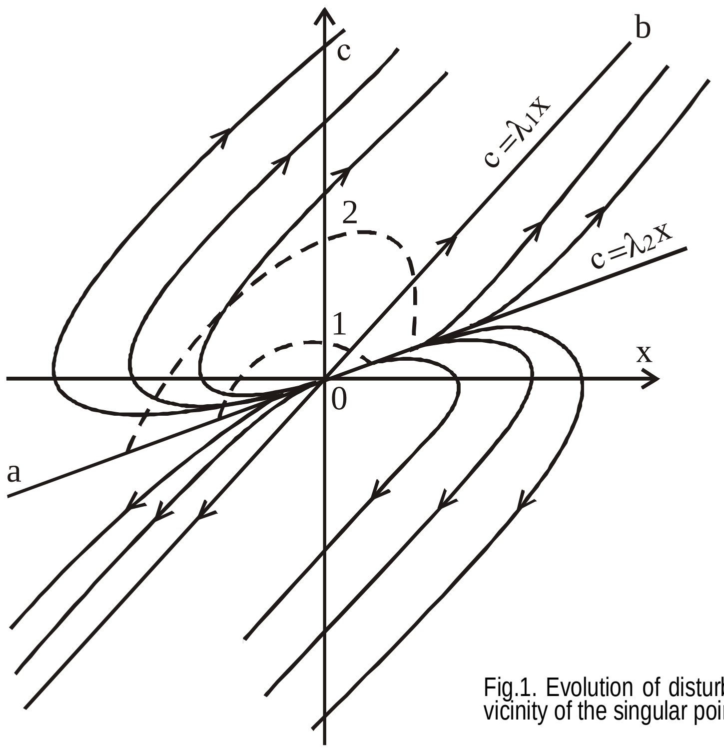

Fig.1. Evolution of disturbance (dash line) of transonic steady solution in the vicinity of the singular point of node type with positive eigendirections

Growth of disturbances will result in establishment of new steady solution, in which sign of *c* does not change in the considered domain and coincides with the sign of the initial disturbance.

2. In the case $l_2 < l_1 < 0$, a singular point is a node with negative eigendirections (fig.2).

When such singularity exists, the area of any bounded in space disturbance tends to zero, and its shape tends to triangular. Hence all bounded in space disturbances attenuate. And leading and falling edges of the disturbance move towards the critical point.

If a constant value of disturbance is maintained at the boundary of the considered domain, new steady solution is established at $t \to \infty$, which satisfies this boundary condition. If new boundary value of *c* given at $x = x_1$ meets

$l_2 x_1 < c(x_1) < 0$,

then continuous solution will appear, which passes through a singular point.

A solution with a shock wave close to the origin appears at $c(x_1) < l_2 x_1$.

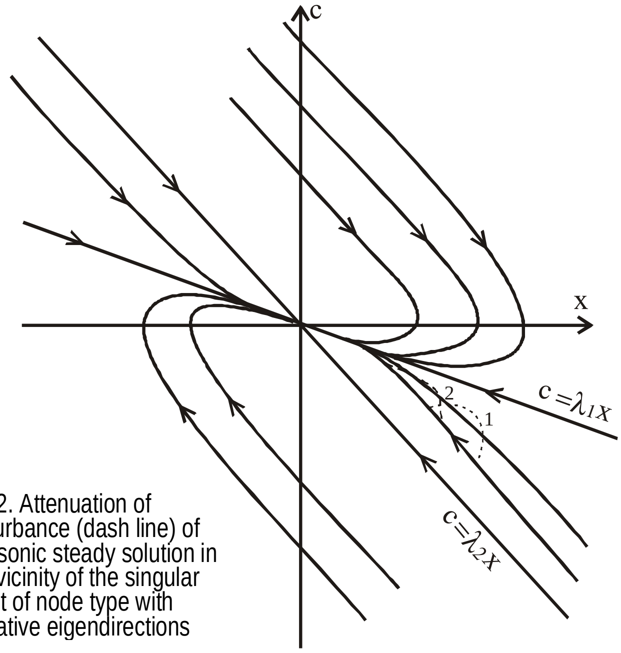

Fig.2. Attenuation of disturbance (dash line) of transonic steady solution in the vicinity of the singular point of node type with negative eigendirections

3. In the case $l_2 < 0$, $l_2 > 0$, a singular point is a saddle (fig.3, 4). Four types of solutions passing through the critical point are possible. These solutions are demonstrated by integral curves *aob*, *lof*, *aof*, *lob* in fig.3.

Consider the disturbance of solution *aob*. Δc value tends to zero like $\exp l_2 t$. Leading and trailing edges of the disturbance move away from the critical point. Disturbance of solution *aob* for to successive moments of time 1 and 2 is shown in fig.3 by dash-and-dot line.

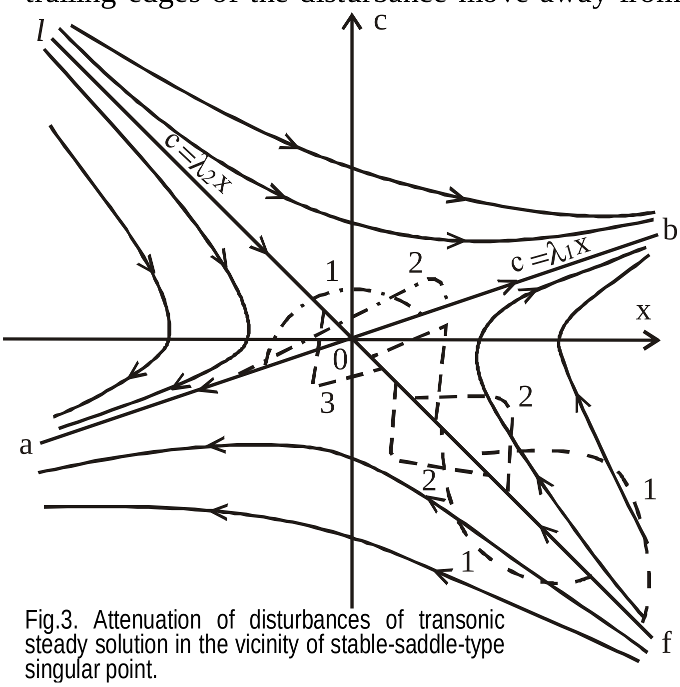

Fig.3. Attenuation of disturbances of transonic steady solution in the vicinity of stable-saddle-type singular point.

Disturbance of solution *lof* converges to the critical point. After rather long period of time, any bounded in space disturbance acquires triangular shape, one side of the triangle belongs to *lof* line, the second side is parallel to *aob*, the third one is parallel to *y*-axis and is a shock wave. The side of triangle, which is parallel to *aob* line, tends to coincidence with this curve in the course of time. The area of disturbance increases at $a > 0$ and decreases at $a < 0$. In the last case, any bounded in space disturbance tends to zero, and its leading and trailing edges move towards the critical point.

Positive and negative disturbances of solution *lof* for successive moments of time $t_1 < t_2 < t_3$...... are shown by dash line in fig.4 for $a < 0$.

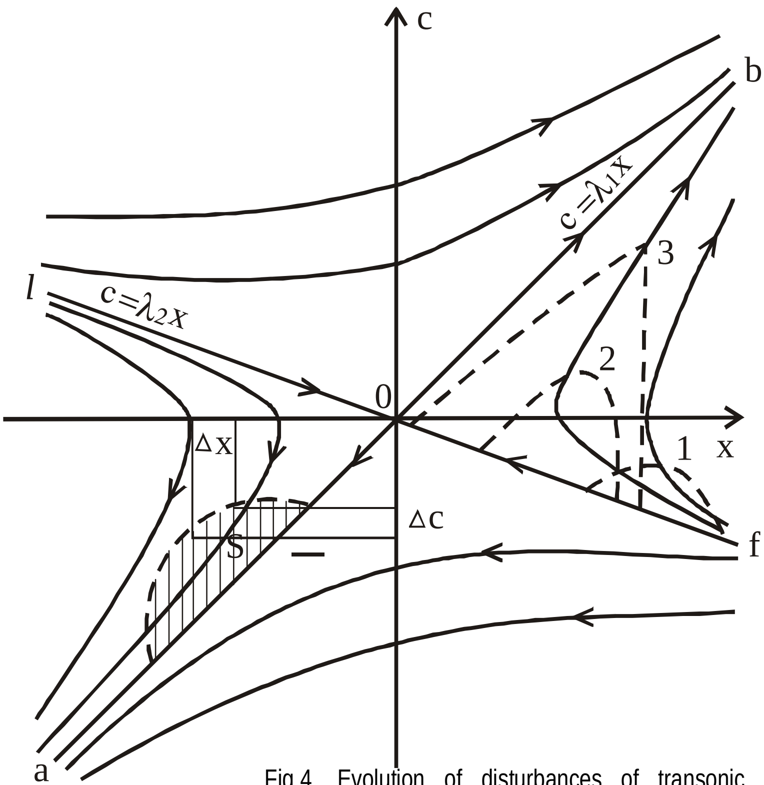

Fig.4. Evolution of disturbances of transonic steady solution in the vicinity of unstable singular point.

If $a > 0$, then the evolution of initial disturbance leads to reorganization of steady solution *lof*. Disturbances with positive *dc* lead to the emergence of a shock wave moving to the right away from the critical point. In this case solution *lob* is established behind the shock wave. Disturbances with negative *dc* lead to the emergence of solution *aof*. If the initial disturbance contains *dc* of both signs, then solution *aob* is established. Positive and negative disturbances of solution *lof* at $a>0$ for the moments $t_1 < t_2 < t_3$ are shown in fig.5 by a dash line.

If a constant value of disturbance *dc* is maintained at the boundary of the considered domain since some moment of time, then at $a < 0$ steady solution containing a shock wave is established in the course of time. The smaller is the boundary value of *dc*, the nearer is the shock wave to the critical point. For this case Fig.5 shows disturbances with positive *dc* for successive moments of time $t_1 < t_2 < t_3$ .....

The disturbance of solution *lob* with positive *dc* attenuates in the same manner as positive disturbance of solution *aob*. Solutions with negative *dc* develop like negative disturbances of solution *lof*. In the case $a > 0$ this leads to the establishment of solution *aob*.

Disturbances of solution *aof* with negative *dc* attenuate like negative disturbances of solution *aob*, and disturbances with positive *dc* develop like the same disturbances of solution *lof*, and at $a > 0$ they lead to the establishment of solution *aob*. Therefore the solution *aob* is always stable in the

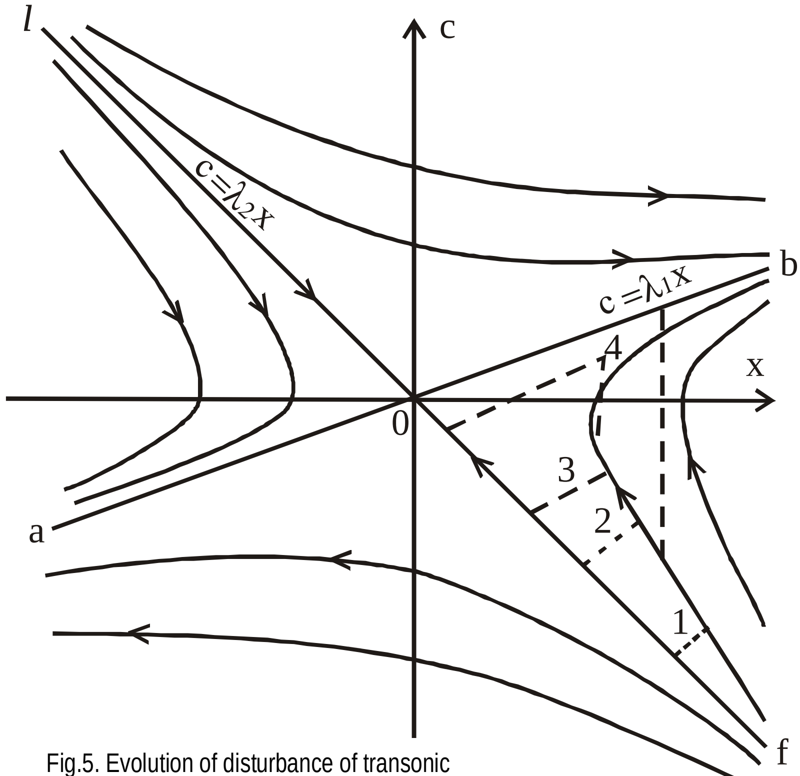

Fig.5. Evolution of disturbance of transonic steady solution when maintaining the disturbance at the boundary

considered sense, and solutions *lof, lob* and *aof* are stable at $a < 0$ and unstable at $a > 0$.
Thus, stability of solution in the vicinity of the critical point depends on the behavior of singularity of steady solutions at this point and is defined by the signs of coefficients a and b. Any solution is stable in the vicinity of the critical point at $a < 0$. At $a > 0$ all the steady solutions in which $c(x)$ turns to zero are unstable, except the solution which is represented by an integral curve passing through a singular saddle-like point with a positive value.

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Minimum relative entropy method for inverse electrocardiography problem

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The usual goal in inverse electrocardiography (ECG) is to reconstruct cardiac electrical sources from body surface potentials and a mathematical model that relates the sources to the measurements [1-20]. Due to attenuation and smoothing that occurs in the thorax, the inverse ECG problem is ill-posed. Minimum relative entropy (MRE) is an approach for inferring a probability density function (pdf), from a set of constraints and prior information. It can be used to solve linear inverse ECG problem. The model is composed of an unknown model parameter vector (epicardial potentials), a measurement (torso potentials) vector, and noise vector in the measurements. The unknown model parameter is considered as a random vector in MRE approach and the solution is given as an expected value of model parameter. The prior information on lower, upper, and expected values of model parameter and the expected uncertainty are the parameters that can affect the output of MRE method. This paper investigates how these parameters can influence the accuracy of the results when the inverse problem is solved using the MRE method. The paper ends with a conclusion and an outlook to future studies.

1. Introduction

Electrocardiography is recording of electric potentials at the body surface that are generated by the electrical activity of the heart. However, in standard electrocardiograms (ECG), only 12 leads are used to characterize the electro-physiological cardiac events, and due to low spatial resolution of these recordings, significant details may be missed. Furthermore, attenuation and smoothing within the torso volume makes it difficult to interpret actual electrical activity in the heart. As an alternative, body surface potential measurements (BSPMs) are recorded from a large number of locations over the body, and estimate cardiaelectrical activity from these dense measurements [1]. This is called as the solution to the inverse problem of ECG.

The inverse problem in electrocardiography is, when the electrical potentials at the body surface are given at all times during a cardiac cycle, to determine the epicardial potential distribution at each instant in time and the activation of isochrones on heart surface, etc., [2, 3]. This article focuses on the inverse solution in terms of epicardial potentials, i.e., the potentials defined on the heart surface with respect to a remote reference electrode. Here, the goal is to figure out the epicardial potential distribution from a given set of body surface potentials and a mathematical model of the torso.

The theoretical relationship between the body surface potentials and the epicardial potentials is given by the linear equation:

$$\mathbf{y}_k = \mathbf{A}\mathbf{x}_k + \boldsymbol{\eta}_k \qquad \left(k = 1, 2, \mathbf{K}, T\right).$$

where $\mathbf{y}_k \in \Re^{M\times 1}$, $\mathbf{x}_k \in \Re^{N\times 1}$ are body surface measurements and epicardial potentials respectively belonging to time t_k and $\mathbf{A}_k \in \Re^{M\times N}$ is the forward transfer matrix. Here, $\boldsymbol{\eta}_k \in \Re^{M\times 1}$ is also added to model to represent discretization errors and noise in the measurements. In this study, we have solved linear inverse ECG problem at each time instant independently, thus we can drop the time index for ease of representation, keeping in mind that the solution should be obtained at every time instant.

Because of attenuation, spatial smoothing and discretization effects, the inverse problem is ill posed; i.e., small perturbations in the measured data can lead an unstable solution and produce unacceptably large errors if it is solved by standard linear least-squares minimization approaches. Regularization methods are the way to cope with the stability problem.

The most popular regularization method, which can be applied to the inverse problem, may be the Tikhonov regularization. The main idea in Tikhonov's method is to include a priori assumptions about the size and the smoothness of the desired solution [5]. However, standard regularization solutions to the inverse problem of electrocardiography have achieved only limited success; main features of epicardial potential distributions are roughly reconstructed using more traditional methods [1]. Oster and Rudy [6] proposed to use prior knowledge of epicardial potentials and used a modified version of Tikhonov regularization called as the Twomey technique to improve the inverse solution. Brooks [5] proposed an approach that combines both temporal and spatial constraints called the joint time/space (JTS) regularization method in order to moderate the bias toward the chosen constraint in the Tikhonov regularization. More recently, statistical techniques such as Bayesian maximum a posteriori (MAP) estimation [19] and Kalman filtering [13] have been used for solving the inverse ECG problem. But in these approaches, one needs to accurately define the prior statistical properties of the epicardial potentials, which is not a straightforward task. The approach used in this paper is based on Minimum Relative Entropy (MRE) method previously presented in [7, 8, 10, 11]. We show that this method can be utilized to solve the linear inverse electrocardiography problem. Furthermore, the prior probability densities are defined as part of the MRE algorithm, which makes this approach easier to use than Bayesian MAP estimation and Kalman filtering approaches.

2. Minimum relative entropy theory

The relative entropy principle is a general, information theoretic method for problem solving. Suppose an unknown multivariate density function of random variable $\mathbf{x}$ is $q^{\dagger}(\mathbf{x})$, which $q^{\dagger}$ consist of initial estimate $p(\mathbf{x})$ and some additional constraint that restricts $q^{\dagger}(\mathbf{x})$, to a specified convex set of probability densities [7, 8]). Typical constraint information includes:

$$q^{\dagger}(\mathbf{x})\geq 0$$

$$\int_{\Re^N} q^{\dagger}(\mathbf{x})dx = 1$$

$$\int_{\Re^N} q^{\dagger}(\mathbf{x})f_k(\mathbf{x})dx = \hat{f}_k, \qquad k = 1,2,\ldots M$$

Then the relative entropy principle states that the estimate $q(\mathbf{x})$ of the probability density function (pdf) $q^{\dagger}(\mathbf{x})$ minimizes subject to constraints as follows:

$$H(q,p) = \min_{q\dagger} H(q^{\dagger},p)$$

Here $H(q, p)$ is the entropy of $q(\mathbf{x})$ relative to $p(\mathbf{x})$, also known as the **Kullback-Leibler divergence**, defined as [20]:

$$H(p,q) = \int_{\Re^N} q(\mathbf{x})\ln\frac{q(\mathbf{x})}{p(\mathbf{x})}dx$$

Furthermore, the constraint given above is extended to incorporate uncertainty about the values of $\hat{f}_k$ to fit the solution within a specified tolerance.

$$\sum_{k=1}^{M}\left(\int_{\Re^N} q^{\dagger}(\mathbf{x}) f_k(\mathbf{x}) dx - \hat{f}_k\right)^2 = e^2$$

One of the standard methods to solve the minimization problem defined above is to introduce Lagrange multipliers to the corresponding the constraints. By introducing the Lagrange multipliers, optimization problem can be written as:

$$q = \arg\min_{q}\left(\int_{\Re^N} q(\mathbf{x}) \ln\frac{q(\mathbf{x})}{p(\mathbf{x})} dx + m\left[\int_{\Re^N} q^{\dagger}(\mathbf{x}) dx - 1\right] + t\left[\sum_{k=1}^{M}\left(\int_{\Re^N} q(\mathbf{x}) f_k(\mathbf{x}) dx - \hat{f}_k\right)^2 - e^2\right]\right)$$

The solution is given as

$$q(\mathbf{x}) = p(\mathbf{x}) e^{\left(-1-m-\sum_{k=1}^{M} l_k f_k(\mathbf{x})\right)}$$

$$l_k = 2t\int_{\Re^N} q(\mathbf{x}) f_k(\mathbf{x}) d\mathbf{x}, \qquad k=1, 2,\ldots, \mathrm{M}$$

The detailed derivation of the equations of minimum relative entropy method can be found in the references [7, 8, 9, 10, 11].

3. Experimental results

The epicardial potentials used in this study are recorded at University of Utah Nora Eccles Harrison Cardiovascular Research and Training Institute (CVRTI) from dog heart that is placed in a realistic torso tank. Then, real epicardial potentials were recorded via sock electrodes from all over the surface of the ventricles. Simulated body surface potentials were obtained by multiplying these epicardial potentials by a forward transformation matrix (obtained using the Boundary Element Method), and adding independent and identically distributed white Gaussian noise to the torso potentials at a signal-to-noise ratio of 30*dB*.

A minimum relative entropy method requires having a prior knowledge about the bounds, the mean value of state variable **x** and expected uncertainty in the measurements. Our aim was to examine the influence of the each prior information to the solution, independent from the others. The procedures followed during our study to investigate the impact of each parameter of MRE method are:

Prior expected value: In order to analyze how the prior expected value influences the output of MRE method, we used fixed upper, lower bounds and expected uncertainty to remove undesired effects causing from change in these parameters. Thus expected uncertainty e^2 was taken as true error inserted to the body surface.

$$e = \frac{\|\mathbf{y} - \mathbf{A}\mathbf{x}\|}{\sqrt{M}}$$

The results are presented in Figure 1. In this figure, and the upcoming figures, we plot the true and estimated epicardial potentials at a single electrode on the heart surface with respect to time. A good estimate should be close to the true epicardial potential signal. In Figure 1, the solutions that are obtained using underestimated and overestimated prior expected values are represented by dashed and dash-dot line, respectively. Furthermore, true epicardial potential and Tikhonov solution as a reference solution are also shown in the same figure with solid line and dotted line, respectively.

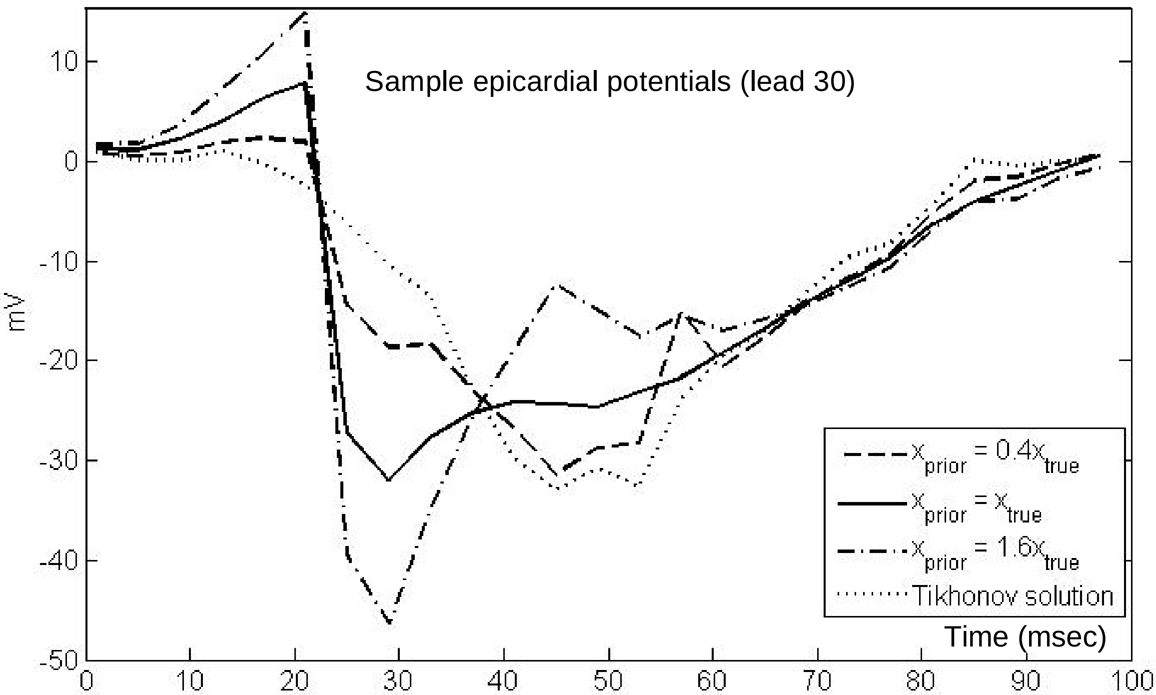

Figure 1. Sample solution of epicardial potentials obtained using underestimated and overestimated prior expected values.

Upper and Lower Bounds: The impact of upper and lower bounds of the state variable (epicardial potential in ECG) on MRE method were investigated by fixing prior expected value and expected uncertainty to their true values. Then we solved the inverse problem for different values of the upper and lower bounds. The results are represented in Figure 2. The obtained solutions using underestimated and overestimated upper and lower bounds are represented by dashed and dash-dot lines, respectively. Moreover, solid and dotted lines shows the true epicardial potentials and the Tikhonov solution.

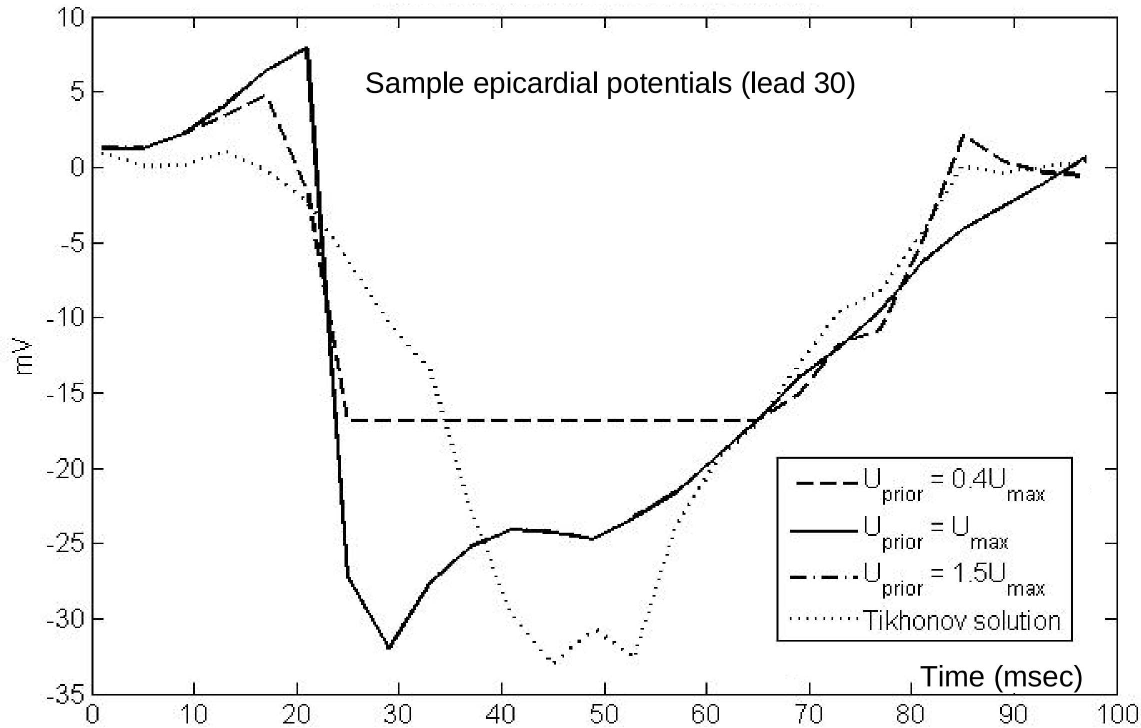

Figure 2. Sample solution of epicardial potentials obtained using underestimated and overestimated upper-lower bounds.

Expected Uncertainty: The upper and lower bounds of the prior epicardial potentials were fixed for each time instant. In addition, prior expected values were set to their true values. Then, using underestimated and overestimated uncertainty parameter values, changes in the inverse solution were recorded. The obtained solutions using underestimated and overestimated prior expected uncertainty are represented by dashed and dash-dot lines. In addition, the Tikhonov solution and true epicardial potential are represented by dotted and solid line, respectively, in Figure 3.

4. Conclusion

This study was designed to figure out how the parameters of MRE can change the final solution in inverse ECG problem. The effects of three parameters were investigated by changing only one parameter and fixing all others in each test case.

- Estimation gets worse if the prior expected value is different from the true epicardial potential. As the prior expected value become more different then the true value, the estimation quality decreases.
- If prior bounds are big enough, which the maximum and minimum value of true epicardial potential can lie between these values, then we did not observed any significant effect on the estimation. But in the other case, the estimation is significantly restricted by given prior lower and upper bounds (see Figure 2).

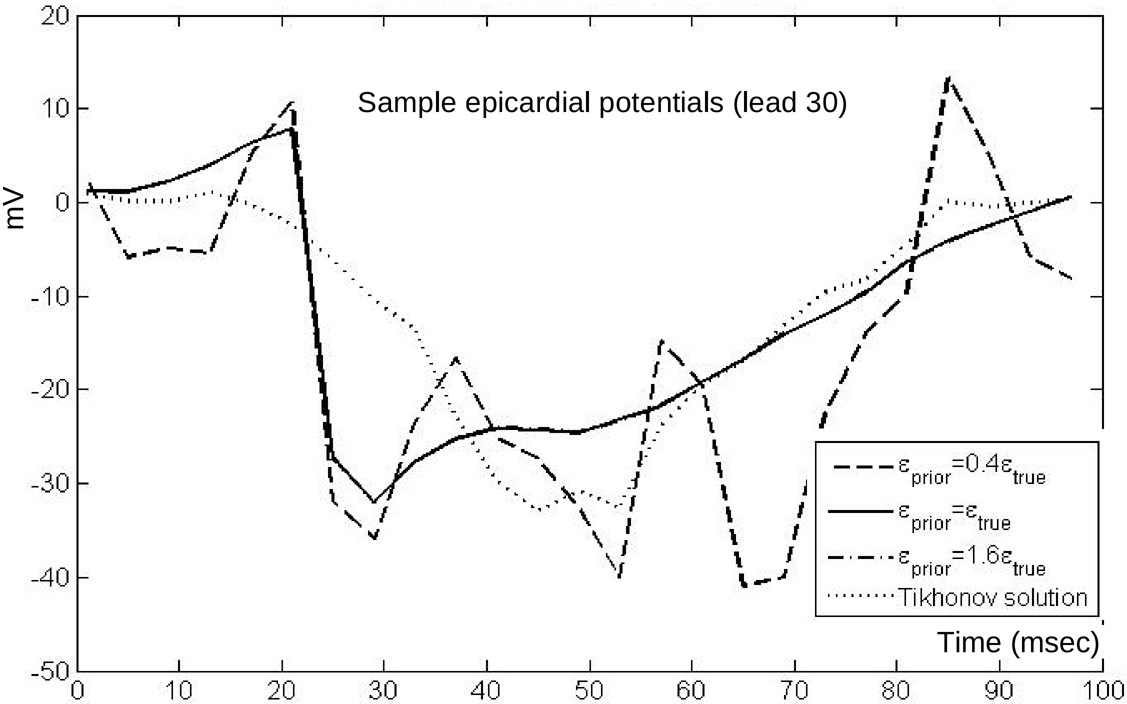

Figure 3. Sample solution of epicardial potentials obtained using underestimated and overestimated prior expected uncertainty.

- Underestimated expected uncertainty is causing considerable degradation in MRE output.

This article demonstrates the importance of modern optimization theory in an important and emerging optimization field. The readers are welcome to join the dynamics of this research.

5. Future Works

CMARS (Conic Multivariate Adaptive Regression Splines) is a nonparametric regression modeling technique that makes no assumption about the underlying functional relationship between the dependent and independent variables, based on the methodology MARS from statistical learning and supported by optimization theory. In recent years, MARS, CMARS and its robust version RCMARS have been successfully applied to many areas of science and technology Refs. [17, 18].

We have seen that the solution of the linear inverse ECG problem using MRE approach is very sensitive to the prior information of parameters, upper, lower bounds, expected uncertainty and prior expected values. As a future study, we will apply (C)MARS techniques to solve inverse ECG problem to overcome this sensitivity problem on parameters which we experienced in MRE method. In addition, as a future work, (C)MARS can be used to solve the nonlinear structural inverse ECG model.

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To dark energy theory from a Cosserat-like model of spacetime

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In Euclidean as well as Riemannian geometry a point cannot rotate. Only a finite length line could rotate. This is where most of the troubles of the general theory of relativity reside. Once realized, the situation could be resolved by going in the direction of Cartan-Einstein spacetime but all the way without stopping. The present work combines the mental picture afforded by Cosserat micropolar spacetime with that of Cartan-Einstein spacetime as well as the Cantorian-fractal spacetime proposal and in the course of doing that, resolves the major problem of dark energy. Various methods are used to validate our main results including 't Hooft-Veltman renormalization method [1-121].

1. General Introduction

Elastic cylindrical shells when pinched in the middle deform in the following slightly unexpected way: Locally in the vicinity of the pinching, the circular cross-section deforms to an oval shaped one. With increasing distance from the pinching region the oval cross-section rotates until it becomes perpendicular to the oval at the pinched middle of the cylinder (fig. 1). This kind of deformation is only possible because of the 'material' nature of the cylinder and is a consequence of the continuum mechanics of a tangible material surface as opposed to an idealized purely geometrical non-materialistic space like those theoreticized by Euclid and Riemann. Cylindrical shells are real structures and as such are endowed with complex shear and torsion forces as is well known from the theory of elasticity, plasticity and Rheology. Likening the local curvature in the pinched region with a positive attractive gravity pulling things together we are logically justified to liken the perpendicular curvature at the extremity of the cylinder with a negative repulsive gravity which push things apart.

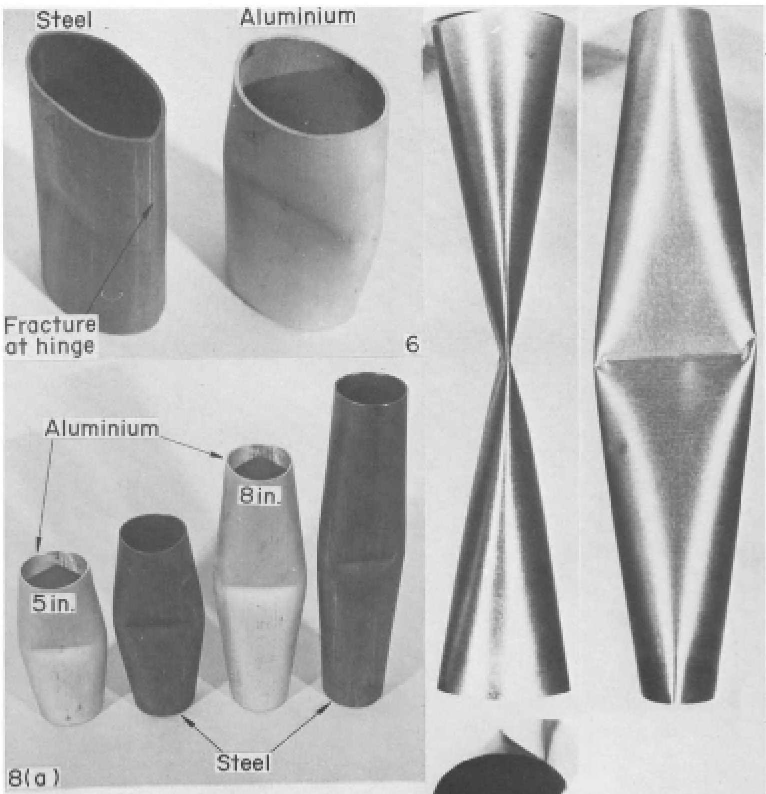

Fig. 1. Actual experiments with elastic and plastic cylindrical shells [74]. The deformation of the pinched cylinder provide an instructive insight into the difference between real material space and abstract mathematical idealization of space. In a real space a local change of curvature at the middle of the pinched cylinder induces a seemingly opposite change of curvature at the edges of the shells. We imagine the situation in 4D space to be a higher dimensional analogue to gravity and anti-gravity.

The present work is concerned exclusively with theoretical physics and cosmology of space, time, matter as well as the quanta of ordinary energy [1-8] and of meta energy, i.e. dark energy [9-28]. The simple analogy between gravity and the deformation of an elastic shell outlined above is taken literally and pushed to its ultimate by imaging the whole set up taking place in four dimensional space akin to that of Einstein’s general relativity but with some additional elements due to Cosserat and Cartan as well as *f*(*T*) gravity, pure gravity, Rindler spacetime, relativistic hydrodynamics, elasticity, plasticity and transfinite *E*-infinity Cantorian spacetime [29-116]. To tame the involved ‘infinitely’ long 4*D* ‘quasi cylinder’ we use the sophistication of hyperbolic geometry and utilize the Poincare-Beltrami projection to establish a connection to a Penrose-like fractal tiling universe [7-9, 29-77]. It is then not particularly difficult to imagine what one will discover next when connecting each of the ramified fractal tiles to a hyperbolic fractal Rindler space (fig. 2). At the circular horizon of the Poincare-Beltrami projection and taking the isomorphic length into consideration, each fractal point is a head of a Rindler wedge [9, 17, 29]. In turn the wedge consists of two parts, a hyperbolic triangle with a ‘topological’ area equal to $f^5/2$ where $f=(\sqrt{5}-1)/2$ and a circular segment shape joined to the triangle with an area or rather a topological measure equal to $1-(f^5/2)=5f^2/2$ [9, 17, 29]. Subsequently we use various facts connected to the thermo-dynamical interpretation of gravity [33], Hawking’s radiation [3, 15, 36, 51], non-commutative geometry, Cantorian *E*-infinity theory as well as the algebraic topology theory of cosmic defects [41] to reason that $f^5/2$ which corresponds to a five dimensional zero point is stemming from three field theoretical dimensions of pure gravity, as it is given by the Vierbien representation $D = d(d - 3)/2$. This gives us that COBE, WMAP and Planck measured 4.5% energy density of the cosmos $E(O) = (f^5/2)(mc^2) = mc^2/22$ where $5f^2/2$ corresponds to a five dimensional empty set stemming from two field theoretical dimensions of pure gravity. On the other hand the 95.5% factor of the ‘missing’ dark energy density $E(D) = (5f^2/2)(mc^2)$ corresponds to the antigravity effect behind the observed accelerated expansion of the cosmos [7, 9, 20, 23]. In this sense and by setting space, time and matter truly on the very same footing we could loosely say that attractive gravity pinches the ‘material’ spacetime counterpart of Einstein’s gravity and produces the observed puzzling anti-gravity accelerating expansion of the cosmos [36]. In fact it is natural to have negative curvature and thus negative gravity in a Cantorian-fractal

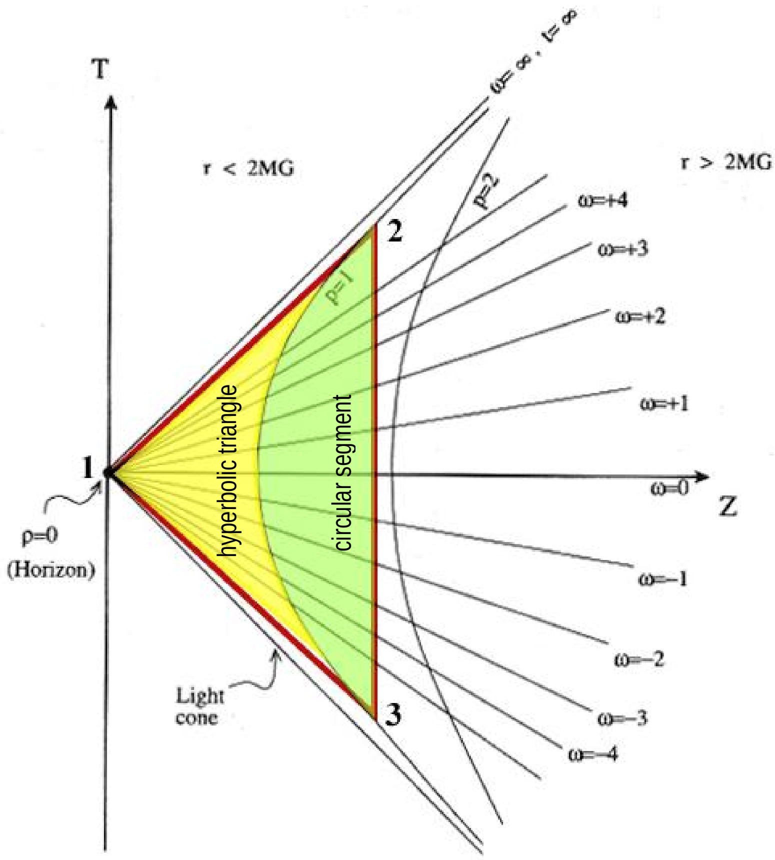

Fig.2. Equal time and proper distance surfaces in Rindler space [9, 17, 29, 30]

Note that the area of the triangle (1, 2, 3) is Lorentz invariant and is given by A= [T(w/2)][(Ƶ w/2)]. In addition the area of the hyperbolic light yellow triangle is equal to the topological ordinary energy (ϕ^5 /2) while the area of the green circular segment is equal to the topological dark energy (5 ϕ^2 /2) where $\phi = 2/(\sqrt{5}+1)$. The actual calculation is given in the main text and depends upon simple integration of hyperbolic trigonometrical functions [31, 79].

spacetime where there are no real points at all and therefore torsion does not vanish by taking the deceptive limit of a fundamentally granular spacetime setting. We conclude this thread by noting that a cosmological constant $L = -1$, a topological empty set dimension $D_T = -1$, negative curvature at a horizon or a conjectured negative dimension of a texture-topological defect (Table 1 of section 9) as well as a field theoretical degree of freedom equal – 1 for pure $2D$ gravity are all but basically tautological statements saying essentially the same thing, namely that there is a cosmic accelerated expansion and that dark energy is what stands behind this negative gravity force [69]. In other words we have various mental pictures and different mathematical formulations however it is still the same empirical reality.

A universal wisdom that has been well tested over the years is that in science as in life, asking the right question is almost half of the answer [1]. It seems that a few scientists were more equipped to ask the right question than most of us and this is the main philosophy probing the present paper [1-14]. Hermann Weyl's famous book "Raum, Zeit, Materie" may be as good a starting point as any [2]. Do we really treat space, time and matter in a democratic way? Sure enough space and time were fused by the Minkowsky-Einstein program however nothing similar was systematically undertaken with the same vigor regarding matter and a somewhat naive materialism prevails in physics [3-5]. For instance the geometry of spacetime used in physics is nowhere taken to be as "real" as the geometry used in say the theory of elasticity or plasticity [25, 60, 62, 113]. Of course there are many models used in relativistic quantum physics which utilized hydrodynamical paradigms and even modified fluid mechanics equations but these important efforts are relatively the exception and do not go as far as one could imagine [24]. To achieve our goal, i.e. to put spacetime and matter on the same footing requires a new material-like geometry [6-28] with granular structure for which the torsional part of the connection [24] does not vanish. Thus finding this material-like geometry [25] is paramount.

In the present work we advocate among other things the idea that such geometry exists since a relatively long time and that it is a generalization of what E.Cartan [101, 102] and the brothers Cosserat developed in 1909 [78] when married to modern Cantorian fractals [7, 8,12,14]. In fact we will show various completely unsuspected relations between metal forming engineering problems and the negative pressure behind the observed unexpected acceleration rather than deceleration of cosmic expansion [15-17]. Said succinctly in a few sentences, we will show that anti-gravity is essentially the same phenomena as anti-curvature of a pinched long cylindrical shell once this cylinder is put in the projective hyperbolic plane corresponding to 4 and 5 dimensional fractal spacetime [7, 8, 12, 14, 66]. Incredible as it may seem at first sight, this is essentially the same thing as saying that dark energy comes from pure gravity as well as the equivalent massless graviton field theoretical $D = d(d-3)/2$ degrees of freedom where d is the dimension of the space of gravity which are in turn related to Weyl tensor and the empty set in 5 dimensional Kaluza-Klein spacetime as well as the representation of the Vierbein discussed earlier on. The present work is thus a monolithic synthesis of the work of Einstein, Cosserat, Cartan, Hawking, Rindler, Penrose, Conne, Unruh, 't Hooft and the school of fractal Cantorian spacetime to mention only a few of the main sources pouring into the present work [15-78]. In addition we will look carefully at the role of the Killing-Yang tensor in explaining negative energy all apart from an instructive analogy between dark energy and capillary forces of hydrodynamics as well as Koiter's theory of imperfection sensitivity of elastically buckled shells [25, 112, 113]. Finally our main results are validated using 't Hooft-Veltman renormalization [119, 120].

2. Keeping an open mind about the fractal-thermodynamical fluctuation origin of gravity

It is important that we point out from the outset a few fundamental points which represent some departure from the orthodoxy of general relativity. In short this requires what we consider a minimum of liberal open mindedness about the following admittedly not universally accepted concepts and experimental findings:

2.1. As in Feynman's conjecture extended by the author we will occasionally view gravity as the effect of the passing of fractal time [7, 8, 21, 22].

2.2. We tend to accept that Hawking's radiation, Rindler's wedge and Unruh's temperature are backed by real physics and are by no means mathematical artifacts [9, 17].

2.3. We are firm on the opinion that Hardy's quantum entanglement is real and was experimentally verified. The golden mean to the power of five first found by Hardy as a quantum probability and recognized as such by the author is profound [19].

2.4. The COBE, WMAP and Planck measurements as well as other recent astronomical as well as astrophysical anomalies are real [47, 80] and will not be dismissed here as misinterpretation, faulty calculations or defects of electronic equipment. This is definitely a majority view of scientists worldwide although I must hasten to say that scientific facts have nothing to do with democratic elections.

2.5. The field theoretical concept of the number of degrees of freedom for pure gravity is of fundamental mathematical and physical importance and is related to dimensionality of the zero set, the empty set as well as to the density of dark energy via $D = d(d-3)/2$ of the Vierbein and the massless graviton where d is the dimension [99]. Inserting in D it becomes evident that $D = -1$ for $d = 2$ is a quasi empty set. Now the degrees of freedom of a massless graviton are also given by the same formula showing quantum mechanics at the root of classical relativity [27, 70, 99].

2.6. Dimensional regularization $D - 4 = \hat{I}$ is essentially going into the direction of an effective quantum gravity theory and setting $\hat{I} = k = 2f^5$ leads directly to the exact dark energy density $E(D) = (D)\, mc^2$ where $g(D) = (4 - k)/4 = f^5/2 = (21/22)$ [119, 120].

2.7. ‘t Hooft's dimensional renormalization method is tacitly a statement on the fractal nature of spacetime and implies that gravity correction to the running coupling constants of four dimensional gauge forces interaction can be substantial at both the Planck scale and by duality the cosmic Hubble scale. This is obvious from $E(D) = [(4 - k)/k]\, mc^2 = mc^2\, (21/22)$ [119-120].

At this point it is appropriate to note the work of Padmanabhan [32, 33] as one of the main guiding lights in uncovering the thermodynamical roots of gravity. On the other hand our hyperbolic geometrical fractal conception of spacetime [79] is also at the root of thermodynamics itself as is obvious from the thermal character of Unruh's temperature [9, 17, 27]. The same point of view applies of course to electromagnetism where we are justified in seeing $\bar{\alpha}_o \cong 137$ as by far more fundamental than Newton's constant, the speed of light or Planck's constant. Finally severe discrepancy between measurement and theory is nothing new and is well documented in all situations where the environment is highly unstable such as is the case with the imperfection sensitivity of buckling of elastic shells [112, 113] which was incidentally the Ph.D thesis of the Author and based upon the work of the leading near to legendary Dutch engineering scientist, W.T.Koiter [112, 113].

3. Einstein in Cosserat-Cartan space

The aim of the present section is to demonstrate how easy it is to reformulate and rephrase Einstein's general relativity within the frame work of the theories of Cosserat [78], Cartan and

Yano [100-103] to account for the observed and quite surprising accelerated cosmological expansion of the universe and the concurrent inference that almost 95.5% of the total energy density of the universe seems to be negative dark energy [10-27].
We start from the premise that both Einstein's spacetime and the maximally symmetric Witten's five Branes model leads to the same Lorentzian factor $g = 1$ for the maximal Einstein energy density, $E = g\, mc^2$ where m is the mass and c is the speed of light and will look upon Cartan's affine connection from a Lie symmetry groups view point [96-98]. Never the less the trivial identity

$$g = D^{(4)}/\, D^{(4)} = N_K^{(32)}/\, N_K^{(32)} = 1$$

where g is the Lorentz factor, $D^{(4)} = 4$ and $N_K^{(32)} = (32)(33)/2 = 528$, implies a far more intricate relation than the deceptively harmless appearance transpires. The rationale behind this assertion is that exactly 504 of the 528 particle-like quantum states may be at least heuristically identified as Cartan-like torsional states [90-93]. This could be deduced with relative ease from an educated counting exercise of the quantum states of Heterotic string theory [103]. In the course of doing that it will become clear that the 504 are the internal killing-Yano hidden dimensions of E8E8 exceptional Lie symmetry group of superstrings plus 8 [106-108]:

$$D^{(8)} + \dim \mathrm{E8E8} = 8 + |\mathrm{SO}\ (32)| = 8 + 2\cdot 248 = 8 + 496 = 504.$$

Details of the computation and counting are given lucidly on pages 383-385 of [103]. The 528 killing vector fields on the other hand are interpreted by us here in two ways. First it is the number of components of the killing-Yano conformal tensor [25] and second it is the sum of the dimensions of E8, E7, E6, E5 and E4 [10-12]. Based on its Dynkin diagram E5 is just another name for $|\mathrm{SO}\ (10)| = (10\cdot 9)/2 = 45$. In other words we have [106-108]

$$\sum_{i=5}^{i=8} |\mathrm{E_i}| = |\mathrm{E}_5| + |\mathrm{E}_6| + |\mathrm{E}_7| + |\mathrm{E}_8| = 45 + 78 + 133 + 248 = 504$$

Adding $|\mathrm{E4}| = 24$ where $|\mathrm{E4}|$ is simply another name for $|\mathrm{SU}(5)|$ of GUT unification [104, 105], we see that

$$\sum_{i=4}^{i=8} |\mathrm{E_i}| = 504 + |\mathrm{SU}\ (5)| = 504 + [(5)^2 - 1] = 504 + 24 = 528.$$

In other words we have [106-108]

$$\sum_{i=4}^{i=8} |\mathrm{E_i}| = \mathrm{N_K}^{(32)} = 528.$$

Consequently the number of the Killing components [84] which are related to the purely "ordinary" energy are given by

$$\mathrm{N_K}^{(32)} - \sum_{i=5}^{i=8} |\mathrm{E_i}| = \sum_{i=4}^{i=8} |\mathrm{E_i}| - \sum_{i=5}^{i=8} |\mathrm{E_i}| = 528 - 504 = 24 = \dim \mathrm{SU}(5).$$

It is vital at this point not to confuse dark energy with torsion energy due to the 24 Riemann-Cartan connection components in four dimensions because the concept of torsion has various meanings in string theory compared to Einstein-Cartan theory [102,103]. In the present work the 4.5% cosmically measured ordinary energy density is due to the 24 Yano-Killing tensor components while the conjectured dark energy is due to the 504 rest components known from the spectrum of Heterotic string theory [103].

Two further relevant observations regarding the vital number 24. First it is exactly equal to the number of the gauge bosons of SU(5) GUT unification which adds 12 bosons to the well known and experimentally found 12 bosons of the SU(3) SU(2) U(1) standard model of high energy physics [106-108]. Second the only pure number in the Killing-Yano totally skew symmetric tensor of the well known 5 expression is $e_{\sigma\pi\in\lambda}\, e^{\sigma\pi\in\lambda} = -24$.
Contemplating the situation a little it is not particularly difficult to convince oneself that the Lorentzian factor of Einstein's energy density corresponding to 24 non-trivial Bianchi dentities is the ratio between the 24 and the Witten bulk of 528 maximally symmetric space. Consequently ordinary energy comes with a Lorentzian factor

$$\gamma_o = \frac{\sum_{i=4}^{i=8}|E_i| - \sum_{i=5}^{i=8}|E_i|}{\sum_{i=5}^{i=8}|E_i|} = \frac{528-504}{528} = 1/22$$

and therefore ordinary is given by

$$E(0) = (g_0)mc^2 = mc^2/22$$

exactly as expected from previous analysis. Dark energy on the other hand is squarely connected to the negative energy of the non-vanishing torsional part in the Cartan connection, namely the 504 known also from the particle physics spectroscopy of Heterotic string theory. The corresponding Lorentzian factor is thus

$$g_D = -(504)/(528) = -\sum_{i=5}^{i=8}|E_i| \Big/ \sum_{i=4}^{i=8}|E_i|$$

leading to a dark energy density

$$E(D) = -\ (504)/(528)\ mc^2 = -mc^2\ (21/22).$$

This is exactly the same result which we find when using 't Hooft's dimensional regularization $D - 4 = \hat{I}$ when setting $\hat{I} = k = 2f^2$ and finding an entangled energy density $E(D) = \left(\frac{4-k}{4}\right) = mc^2\ (21/22)$.

Einstein's energy density $E = mc^2$ on the other hand is blind to the preceding fine distinction which wrongly considers 4/4 = 1 completely equivalent to 528/528 = 1, and is therefore given by the sum of the absolute value of both energies as

$$E(\text{Einstein}) = E(0) + |E(D)| = mc^2\left(\frac{1}{22}+\frac{21}{22}\right) = mc^2.$$

In other words Einstein's maximal energy formula does not need to be quantumly corrected but only quantumly dissected into two parts.
Now we could make another profound interpretation of this result if we consider E(Einstein) to be unity by setting $m = c = 1$. That way the dark energy could be viewed as a Legendre transformation of ordinary energy, that is to say it is a complementary energy as far as the absolute value is concerned. In other words, dark energy is the negative value of the complementary energy or the ordinary measurable energy. We note on passing that 528 is divided in Witten's model into 1D strings, 2D membranes and 5D Branes

$$N_K^{(32)} = \binom{11}{1} + \binom{11}{2} + \binom{11}{5} = 11 + 55 + 462 = 528$$

The corresponding E8E8 expression includes the point-like particles as well as the 3*D* and 4*D* Branes

$$\mathrm{N(E8E)} = \binom{11}{0} + \binom{11}{3} + \binom{11}{4} = 1 + 165 + 330 = 496 = |\mathrm{E8E8}|$$

The hidden Yano-Killing 504 on the other hand are given in Heterotic super string theory by three groups of states, namely 480, 16 and 8 leading to

$$480 + 16 = 496 \quad \text{and} \quad 496 + 8 = 504$$

as explained in great detail. We also note that

$\mathrm{N_K}^{(32)} + \mathrm{N(E8E8)} = 1024$ while $d_c^{(11)} = 1024$ for $d_c^{(0)} = (1/2)$ and $\sum_{i=0}^{i=11} \binom{11}{i} = (2)(1024) = 2048$.

Note also that in various Heterotic string theories different divisions exist. For an in depth study of the E-line exceptional Lie symmetry groups at the root of the present theory [30-32] could be considerably helpful.

4. Elementary derivation of Einstein's revised formula for ordinary energy $E \simeq mc^2/22$

Evidently when Einstein o drove his famous $E = mc^2$ he did not write a Lagrangian [24]. However supposed he knew how to do what is according to current prejudice the only acceptable way forward, namely writing down a Lagrangian [99]. First such a Lagrangian would lead in our opinion to a few realizations. For a start the only degree of freedom from a particle physics view point would be the only messenger particle known at the time of Einstein, namely the photon. Energy on the other hand would be the Eigenvalue of a Schrödinger equation however not that of a particle but rather that of the entire universe [24]. Thus E of Einstein would be the Eigenvalue of an unknown quantum gravity Schrödinger-like equation. However we know in the meantime that the physics of our universe is best approximated by at least 12 photon-like particles and not only one photon. On the other hand we know very well that reducing a 12 degrees of freedom Lagrangian to only one degree of freedom Lagrangian would lead to a gross over estimation of the corresponding Eigenvalue, i.e. the energy E of Einstein which is a well known theorem by Lord Rayleigh [25]. Consequently we see that $E = mc^2$ is much larger than what a locally 12 degrees of freedom Lagrangian allows. So much for the qualitative situation. The quantitative one is more involved. None the less an educated guess leads to the following line of reasoning. Noting that the kinetic energy of Newton $E_N = mv^2/2$ and E of Einstein differs mainly by a "scaling" factor (1/2) when disregarding the limit $v \to c$ and noting also that self similarity is a fundamental aspect of both the macro (solar system) and the micro cosmos (Bohr atom) then one is encouraged to think that $E = mc^2$ could be scaled down proportionately to $E = mc^2/22$, where 1/22 is the scaling factor [27]. The value 22 could be thought of in two different obvious ways. It is the 26 bosonic dimensions of the Veneziano spacetime minus Einstein's 4 dimensional spacetime, i.e.

$$l = 1/(26 - 4) = 22,$$

or alternatively we use $E_N = mv^2/2$ and invoke the scaling

$$l = 1/\left[|\mathrm{SU(3)\ SU(2)\ U(1)}| - \gamma\right] = 1/(12 - 1) = 1/11$$

and the limit $v \to c$ to find that [87-89]

$$E_N \to E(\mathrm{O}) = l\left(\frac{1}{2}m(v \to c)^2)\right) = \left(\frac{1}{11}\right)\left(\frac{1}{2}\right)mc^2 = mc^2/22$$

The preceding plausibility derivation could be made mathematically water tight in various ways discussed in previous publications and will not be followed here any further in order not to lose the main thread of the present work and its objective.

5. Details of Rindler space calculations leading to the areas $f^5/2$ and $5f^2/2$ of ordinary and dark energy respectively

We follow Fig. (2) which represents a Rindler space and the associated horizon [17, 29, 30]. In the following analysis we concentrate on the questions pertaining to measure theory, i.e. the various Lorentzian invariant hyperbolic areas and for the moment relegate the question of physical interpretation to a back seat. We see that we have three distinct areas. The first A_o is the total area of the large triangle, H, P, P^1. Calculating the area of A_o is truly trivial since it consists of two symmetric triangles leading to [9, 17]

$$A_o = 2\left[\frac{1}{2}\, t(\eta/2)Ƶ(\eta/2)\right]$$

where h is the opening angle of the Rindler wedge [17, 29, 30]. Since

$$t(h) = \mathbf{l}\ \sinh(h)$$

and

$$Ƶ(h) = \mathbf{l}\cosh(h)$$

We can write A_o as

$$A_o = \mathbf{l}^2 \cosh(h/2)\sinh(h/2)$$

where $\mathbf{l}$ = $1/a$ is the distance between the Rindler horizon and the observer as seen by him and a is the constant Rindler acceleration [29, 30].
The second area is the symmetric hyperbolic segment A_1 as shown in Fig. 2 [20, 30]. This gives twice the integral of half of the segment as

$$A_1 = 2\int_o^{\eta/2} \sinh\left[\mathbf{l}^2 \sinh(h/2)\cosh(h/2) - h/2\,\mathbf{l}^2\right]dh = 5f^2/2$$

Since $Ƶ(h) = \mathbf{l}\cosh(h)$ then $\dfrac{dƵ\ (h)}{dh} = \mathbf{l}\,\mathrm{subg}(h)$ and therefore $d\,Ƶ(h) = \mathbf{l}\sinh(h)\,dh$.

Inserting one finds

$$A_1 = 2\int_o^{\eta/2} t(\eta)\,dƵ\ (\eta) = 2\int_o^{\eta/2}\mathbf{l}\sinh(h)\,\mathbf{l}\sinh(h)\,dh = 2^2\,\mathbf{l}\int_o^{\eta/2}\sinh(h)^2\,dh.$$

This is a straight forward simple integration but could also be found in any standard handbook of mathematics to be

$$A_1 = 2\ \mathbf{l}^2\left[\frac{1}{2}\sinh\eta\cosh\eta - \frac{1}{2}\eta\right]_o^{\eta/2} = \mathbf{l}^2\left[\sinh\eta\cosh\eta - \eta\right]_o^{\eta/2} =$$

$$= \mathbf{l}^2(\sinh h/2)(\cosh h/2) - (h/2)(\mathbf{l}^2).$$

Finally following Fig. 2 the area of the hyperbolic triangle A2 is found simply as the difference between A_o and A_1. Consequently

$$A_2 = A_o - A_1 = \mathbf{l}^2\cosh h/2\,\sinh h/2 - \left[\mathbf{l}^2\ \sinh(\eta/2)\cosh(\eta/2) - \eta/2\,\mathbf{l}^2\right] = (\mathbf{l}^2 h)/2.$$

It is one of the fundamental results of the unit interval "topological" physics introduced in earlier work that $c = f$ and $m = f^3$. On the other hand a few moments of deep reflection will reveal that $m = h$ and $c = \mathbf{l}$. Consequently $h \simeq f^3$ and $\mathbf{l} = f$.
Inserting in A_2 one finds $A_2 = f^5/2$. That immediately leads to our second most important result, namely $A_1 = 5f^2/2$ where $\phi = (\sqrt{5} - 1)/2$. It is a trivial matter to see that rounding the value of A_1 and A_2 to the nearest integer gives us the "exact integer" value of the density factor of ordinary energy $\simeq (1/22) = g_2$ and dark energy $\simeq (21/22) = g_1$. This is the same result of preceding sections.

6. Pinching spacetime

Various experiments with pinched elastic and elastoplastic cylindrical shells were actually performed long ago (Fig. 1) [74]. In fact it is extremely easy to demonstrate the effects of induced local change of curvature causing a considerable distance away a change of curvature of opposite sign [15]. For that we need nothing more than a large sheet of writing paper rolled into a long cylinder and squeeze it in the middle as described in previous work. That way we establish at a minimum an analogy connecting not only engineering metal forming with cosmology but also with thermodynamics. The analogy makes it plausible that local attractive deformation causes anti-attraction far away from the local opposite sign attraction. Curiously the Master Thesis of the Author was about physical nonlinearity of torsion in some elastic structures [114]. That is exactly what is missing in Einstein's geometry and that is exactly what Cosserat and Cartan provide. Also by coincidence of providence the Ph.D of the Author forty years ago was on the effect of imperfection sensitivity on unstable points of bifurcation of elastic shells which is a classical counterpart to quantum wave collapse and missing dark energy [113].

7. Self similarity, P-Adic quantum physics and Cantorian spacetime

Integers are possibly the simplest source of self similarity in physics. A trillion is nothing but unity scaled up a trillion times. Number theory is of course very close to the continuum hypothesis and consequently the most fundamental question regarding the nature of space and time. It is therefore important to understand the intimate relation of the present paper with the fundamental result found by the school of P-Adic quantum physics which we discussed in some details on previous occasions [37, 40]. We stress that only zero and infinity are not ordinary numbers but deep mathematical-philosophical concept. Since unity differs only by a scaling factor we see the fundamental meaning of the unit interval physics [79, 88] where the speed of light is a natural topological quantity $c = f$.

8. Dark energy and dark matter segregated and unified

On the most fundamental level of transfinite set theory we have only the ordinary energy connected to the quantum zero set particle, i.e. $E(\mathrm{O}) = (f^5/2)mc^2$ which is directly proportional to the area of the hyperbolic triangle of the Rindler wedge A_2 and the dark energy connected to the quantum empty set of the wave proportional to the circular segment area $A_1 = 1 - A_2 = 5f^2/2$. Clearly not all of A_1 dark energy is pure energy but some of it is dark matter exactly as part of the ordinary energy is ordinary matter expressed in terms of energy following the theoretical insight of Einstein and the essence of his formula if not its exact quantative prediction which needed the present revision. We are not yet in a position to give a stringent mathematical distinction between dark energy and dark matter, which although lumped together in the energy of the five dimensional empty set theory, has different physical effects and manifestation. However what we can do here is to give a logically coherent plausibility explanation converging towards a mathematical water tight explanation for the difference between dark energy and dark matter [27].

Let us recall first that our previous calculations demonstrated that while 4.5% of the energy density of the cosmos is measurable ordinary energy and matter, the rest, i.e. 100–4.5=95.5% of the energy density must be in the form of dark energy which we believe to be responsible for the initially surprising astrophysical observations connected to the accelerating cosmic expansion in addition to dark matter which we presume to be responsible for various astronomical anamolous observations. Let us further recall that our fundamental equation from which we construct our most fundamental coupling constant, namely $\bar{\alpha}_o \cong 137$ is found from [17, 21, 27]

$$\bar{\alpha}_o = \bar{\alpha}_1\,(1/f)+(\bar{\alpha}_2 = \bar{\alpha}_1/2)+\bar{\alpha}_3+\bar{\alpha}_4 = (60)(1/f)+30+9+1=137+k_o=137+f^5(1-f^5)=137.0820393$$

where $f = (\sqrt{5}-1)/2$ and $\bar{\alpha}_4 = \bar{\alpha}_{QG} = 1$ is the largest possible quantum gravity inverse coupling. The next step in our plausibility "derivation" is to notice that $\bar{\alpha}_1+\bar{\alpha}_2+\bar{\alpha}_3+\bar{\alpha}_4 = 100$ and that this sum could be viewed as a normed value for summing over all the infinite dimensions spanning the fractal-Cantorian spacetime of our theory. In other words this 100 is a normed value for the number of internal as well as external dimensions or brocken symmetries. Now we divide these dimensions into three categories. First the "visible" dimenhsion, i.e. the 3 space dimension plus the time dimension of our classical daily experience. The second category of dimensions are the compactified 22 left from the bosonic Nambu-Veneziano strong interaction dimension. The third category of dimensions are the diluting rest, i.e. 100 – (22 + 4) = 100 – 26 = 74 which represents a finite value for the infinitely many fractal dimensions spanning our fractal spacetime. The next step in our explanation is now quite obvious. We hold it that the various percentages of the energy density of the universe are based on the preceding categorical subdivision of the various normed expectation numbers of the spacetime and internal dimensions. In other words, the four dimensions of spacetime correspond to 3 percent ordinary matter and 1 percent ordinary energy and radiation making up 4% altogether. The 22 compactified dimensions on the other hand correspond to a 22% dark, i.e. "compactified" matter. Finally we are left with the well hidden and diluted rest, namely 100 – (4 + 22) = 74% truly pure dark energy responsible for the negative pressure behind the observed accelerated cosmic expansion. Neddless to say these results, taken on face value, are simple integer approximations of the various cosmological measurements the majority of which put ordinary energy at 4.5% ≈4%, dark matter 22% and dark energy 74.5≈74% [32, 47-51].

The interesting question on the fundamental level of set theory is to ask how the empty set splits into two sets separating pure dark matter from pure dark energy. Our guess is that it is a very similar phenomenon and analysis to that leading to phase transition from purely ordinary matter to purely ordinary energy [24]. The set theoretical analysis behind the preceding illucidation is currently in progress but we decided to release the present incomplete information in the hope of attracting more thinking in this direction.

9. Topological defects, texture and the empty set

Table 1

Topological defect	Dimension
Domain walls	2
Strings	1
Monopoles	0
Textures	–1 (conjectured)

An extremely powerful mathematical subject which benefited cosmology is understandably algebraic topology. Without going into any detail we note the information given in the following Table (1) and add the conjecture that the dimension of texture is –1 and that it could be extrapolated to mean an empty set-like Cantorian wild topology akin to

Alexander Horns. Texture in this interpretation corresponds to dark energy and the negative sign to a negative cosmological constant [52].
Cosmic topological defects following the classification of Vilenkin and Shellard [52]. Note that we added the conjecture dim(textures) = –1 which in effect equates texture to an empty Cantor set.

10. The fundamental role of Hardy's quantum entanglement

The fundamental importance of the theoretical discovery of Hardy's probability of quantum entanglement $P(\text{Hardy}) = f^5$ and its subsequent accurate experimental verification cannot be stressed enough. At a minimum the present work and the understanding of the essence and meaning of dark energy could not be understood in its full ramifications without the quantum entanglement of the cosmos. Without repeating previous arguments and analysis, we just recall for the sake of completeness that $E(0)$ and $E(D)$ could be interpreted and written in terms of Hardy's quantum entanglement as

$$E(0) = P(\text{Hardy})\left(\frac{1}{2}m(v\to c)^2\right) = (f^5/2)\, mc^2$$

and

$$E(D) = 1 - (E(0) = (5f^5/2)\, mc^2).$$

11. Intermediate discussion

In a sense we are dealing here with a cosine of the butterfly effect. A small feeble local effect in the form of an attractive gravity induces at infinity an accumulated effect of anti-gravity adjacent to the horizon. In a sense exotic ideas that the universe may resemble a giant black hole or a knot complement at infinity and therefore neither open nor closed by topologically clopen may not be that far off after all. That way cosmological data and observation collected over a very long period culminating in several Nobel Prizes in Physics has fused various theories together and confirmed the reality of Hawking's radiation, Unruh's temperature, anti-gravity and Rindler spacetime all apart from completing the magnificent work of Einstein's relativity, Planck-Bohr-Heisenberg's quantum mechanics and Boltzmann thermodynamics as indicated in the eminent work of T.Padmanabhan and his school [32, 33]. One could of course argue that the part of the present derivation which is based on an analogy between metal forming, pinching of elastic tapes and the real behavior of a material spacetime micropolar elasticity is less fundamental than previous derivations starting from the zero set as a pre-quantum particle and the empty set as a pre-quantum wave [36-38]. However this is a largely subjective judgment and a matter of taste and personal philosophical stance. In fact one could view the difference between the negative dimension of the empty set $D_T = -1$ as well as the $D = -1$ degree of freedom of pure gravity for $d = 2$ and the cosmological constant $L = -1$ as mere mathematical and physical tautology. We have to admit that because of space limitation we have hardly touched upon many other vital points which could have enhanced understanding the magnificent interconnectivity of mathematics, high energy physics and cosmology leading to the present synthesis. For instance we did not discuss the role of symplectic geometry [66] which would have made the appearance of the golden mean and its derivatives and powers everywhere in our theory plausible, even unavoidable. However the reader may find all these points and more adequately covered in [63] to [70].

12. Capillary surface energy elucidation of the cosmic dark energy – ordinary energy duality

This short section reports on an unsuspected and quite surprising connection between capillary forces and dark energy. As the reader realized from the previous sections and as is

evident from numerous previous publications a fundamental theory was advanced to explain the baffling cosmic observation associated with conjectured dark energy and the surprising measured accelerated rather than decelerating expansion of the universe. Our most rigorous theory was an exact calculation based on particle-wave duality in highly mathematical set theoretical formulation led to an ordinary measurable energy density of $E(0) = mc^2/22$ where m is the mass and c is the speed of light, i.e. –
only 1/22 of Einstein's famous energy density [4]. This was a remarkable result and in full agreement with the latest and most accurate cosmic measurements and supernova analysis which led to the award of several Nobel Prizes in Physics on two different occasions. For dark energy the density found and reconsidered here was $E(D) = mc^2$ (21/22) which amounts to exactly 1– $E(0)$ showing with absolute clarity that Einstein's density, lacking the quantum component, is blind to any distinction between ordinary energy and dark energy. Thus apart from the quantative resolution of this major problem, a fundamental conclusion was reached elevating Einstein's relativity formula $E = mc^2$ to a quantum relativity equation $E = (mc^2/22) + mc^2\ (21/22) = mc^2$ where $E(0)$ is the ordinary energy of a quantum pre-particle in a five dimensional Kaluza-Klein spacetime and $E(D)$ is the negative dark energy of the quantum pre-wave in the same Kaluza-Klein spacetime [34]. Seen in this way we begin to understand why ordinary positive energy can be detected and measured while the negative dark energy could not, at least not directly nor using any conventional method. The reason for this failure is as simple as it is unexpected and is anchored in the deep logic of set theory. A quantum particle is in theoretical terms a physical materialization of the zero set. The quantum wave on the other hand is the physical materialization of the empty set [35, 36]. Since "measurement" interferes with the empty set and causes it to become non-empty, the empty quantum "wave" set transmutes instantly to a zero quantum "particle" set at measurement. This is what we call wave collapse and that is why the negative dark energy of the wave cannot be measured in the ordinary way unless wave non-demolition measurements could be developed in the future [86].
The preceding set theoretical explanation, although mathematically and logically accessible and in some sense even intuitive, cannot be called physically obvious. For instance it is true that we have a clear picture of a particle with a wave as its cobordism, i.e. as its surface. Never the less particles and surface, although inseparable, cannot be dealt with experimentally except via the contra-intuitive perspective of wave-particle duality. All the same it would be more than desirable to have a conjugate more down to earth and conventional physical picture to go hand in hand with the fundamental set theoretical interpretation just outlined.
In the present work we think that we have at long last found a parallel physical interpretation to our set theoretical picture which is in a one to one correspondence with the zero set-empty set particle-wave duality. This we explain next.
Let us consider a capillary surface [82] which is something well known in fluid mechanics and in fact from various simple experiments which almost everyone encountered in elementary school physics. On a fundamental level however the phenomenon involves very complex nonlinearity effects and is related to the theory of a minimal surface. The point is that the energy on the surface is meta-stable and is susceptible to spontaneous symmetry breaking bifurcation instability by jumping into a much lower energy state similar in principle to phase transition as well as local buckling of thin walled structures, a field in which the present author was initially trained and specialized. As we said earlier the subject is also closely related to minimal surfaces [83] and we note an almost esoteric property of capillary surfaces which is that although real, they have no thickness at all. This is somehow an unexpected bridge between the pure mathematics of transfinite set theory and the real physics of capillary fluid mechanics. We note further that despite the fact of being meta-stable,

capillary surfaces are remarkably persistent in some experiments [82] which make a good analogy to the steady state propagation of a quantum wave.
To sum up we could look upon dark energy which is the negative energy of the quantum wave surface of the quantum particle core as being analogous to the physically and classically real capillary surface energy which cannot always be easily measured due to spontaneous jump into the lower energy level of the core. Here we are speaking of higher and lower in absolute terms and are of course disregarding the sign convention. We conclude by noting the immense importance of relativistic hydrodynamical models in physics and astrophysics [106].

13. Dark energy from pure gravity

In the present section which maybe the most important of the entire paper, we start from the basic concept of pure gravity, i.e. gravity in the total absence of any matter field [107]. The well established relevant equation in this case connecting the field theoretical degrees of freedom D of pure gravity to the dimension of the space d is given by [108, 109]

$$D = d(d - 3)/2.$$

Thus for the fundamental situation of $d = 2$ corresponding in string theory for instance to the string world sheet [117] we have the remarkable negative value $D = -1$ for what we called degrees of freedom [69]. This is formally identical to the Menger-Urysohn topological dimension of the empty set [12, 27]

$$D(\text{empty}) = (D_T, D_H) = (-1, f^2)$$

where D_H is the Hausdorff component and $\phi = (\sqrt{5}+1)$ and conceptually this negative degree of freedom has almost the same essential meaning of an empty set. We should stress again that a negative degree of freedom makes no physical sense at all and little if any mathematical sense that is unless it is understood as an empty set. Further more for the classical case of d= 3 we have obviously a practically zero set $D = 0$ corresponding to [20, 87]

$$D(\text{zero}) = (D_T, D_H) = (0, f).$$

Lifting the Hausdorff dimension component of both the empty "pure" gravity and the "zero" gravity to five dimensional Kaluza-Klein spacetime, we find the following pseudo volume, namely [20, 87]

$$\text{Vol}_{(5)}\ (\text{pure gravity}) = f^2 + f^2 + f^2 + f^2 + f^2 = 5f^2$$

and $$\text{Vol}_{(5)}\ (\text{zero gravity}) = (f)(f)(f)(f)(f) = f^5 = P(\text{Hardy}).$$

Noting the volume interpretation of the Hausdorff dimension our total volume, i.e. that modeling the 5D quantum wave of pure gravity and that modeling the 5D quantum particle of zero gravity, one finds - [20, 21, 87]
Total $\text{Vol}_{(5)} = 5f^2 + f^5 = 2 = \dim(\text{string world sheet})$.
The relative density for pure gravity corresponding to the wave is therefore $g_w = 5f^2/2$ while that corresponding to the particle is clearly $g_p = f^5/2$.
Inserting in Einstein's energy density we find both the ordinary measurable energy

$$E(0) = g_p\, mc^2 = (f^5/2)\, mc^2 \simeq mc^2/22$$

and the "meta" dark energy

$$E(D) = g_w\, mc^2 = (5f^2/2)\, mc^2 = mc^2\ (21/22)$$

exactly as expected [34-36]. Thus the fact that empty Einstein space is so rich in structures and the equality of the degrees of freedom of massless graviton and that of pure gravity indicate that Einstein's relativity is closer to quantum mechanics than we ever imagined [23, 24, 100].

14. Dark energy from 't Hooft's dimensional regularization [119, 120]

Ignoring the effects of gravity at grand unification energy scale is questionable. By contrast ignoring the effects of gravity in the running of coupling constants of gauge forces near the Planck scale is totally wrong [15, 21-24]. By *T*-duality the same is true at the opposite extreme, namely cosmic scales [6-8, 21, 22]. These facts are well known and understood in *E*-infinity Cantorian spacetime theory and without going into detail, we just mention a few important facts due to their importance for the analysis in this section. First the theoretical *E*-infinity electromagnetic fine structure constant could be reconstructed correctly only when $\bar{\alpha}_Q = 1$ of quantum gravity is included [21, 27, 37]

$$\bar{\alpha}_o = (\bar{\alpha}_1)(1/f) + (\bar{\alpha}_2 = \bar{\alpha}_1/2) + \bar{\alpha}_3 + (\bar{\alpha}_4 = \bar{\alpha}_Q) = (60)(1/f) + 30 + 9 + 1 =$$
$$= 137 + k_o = 137.082039325 \simeq 137.$$

Second the Heterotic string dimensional hierarchy starts with $(\bar{\alpha}_o/2)$ multiplied with f^n to generate after 8 steps the following values [120, 121] $42 + k$, $26 + k$, $16 + k$, 10, $6 + k$, $4 - k$, Clearly $4 - k$ where $k = f^3(1-f^3) = 0.18033989$ is the fractal Hausdorff dimension at the corresponding Planck and Hubble scale.

It is remarkable that the preceding Cantorian Weyl-Nottale scale relativity [2, 12, 18, 28] fits seamlessly into the 't Hooft-Veltman dimensional regularization scheme [119, 120]. There we use $4 - D = \hat{I}$ to over come divergence and here we just set $\hat{I} = k = 0.18033989$ to account for the topological entanglement due to Hardy's quantum entanglement P(Hardy) = $f^5 = k/2$ where $f = 2/(1+\sqrt{5})$ [34-36]. It is thus not difficult to see that $E = mc^2$ must be scaled to $(E)\left(\frac{4-k}{4}\right)$ to account for uncorrelated parts of the energy. For the correlated parts of the energy which is the ordinary measurable energy we just need to take the Legendre transform, i.e. the complementary energy of the uncorrelated so called dark energy, namely $1 - \left(\frac{4-k}{4}\right)$.

That way we find the dark energy component

$$E(D) = (mc^2)\left(\frac{4-k}{4}\right) = (mc^2)\left(\frac{21+k}{22+k}\right) \cong (mc^2)\,(21/22)$$

and the ordinary energy component

$$E(0) = 1 - E(D) = (mc^2)\,(21 + k) \cong mc^2/22$$

all in full agreement with previous derivations and cosmic observation and analysis [15-17, 22, 23]. The preceding analysis is effectively saying that $4 - k$ of 't Hooft and Veltman is more than a mathematical trick to extract the correct result and avoid divergence [119, 120]. It is an aspect of physical reality and indicates the fractal nature of spacetime and the role of "gravity" in eliminating some unwanted mathematical problems in the exact renormalization equation of gauge fields [37, 68, 119, 120].

15. Conclusion

The preceding analysis may be seen as a Cosserat-like material spacetime based analysis in the spirit of the analogies discussed in connection with the pinched elastic cylindrical shell (Fig. 1) [23]. We stress that the failure of Einstein's general relativity to predict dark energy directly could easily be explained via the Cosserat-like theory presented here [23, 24]. However first Einstein and then Cartan were aware of the problem and proposed what became known later on as the Teleparallelism theory [24, 27]. Thus our Cosserat-like theory points essentially in the same general direction as the Teleparallelism relativity theory [24, 27]. It should not pass unnoticed that $E = mc^2$ is not simply a final conclusion in the special theory of relativity [24]. It is far more than that [118]. It implies the general theory of relativity and connects it to thermodynamics long before anyone noticed that including Einstein himself. Needles to repeat what W.Rindler stressed in all his writing that $E = mc^2$ is a leap of faith and a visionary step which does not follow directly from the special relativity only. However this giant leap has paid off and was a risk worth taking. Finally we must express our deep satisfaction about the robustness of the result $E = (mc^2/22) + mc^2\ (21/22) = mc^2$ which we always reach using virtually any reasonable theory. The inescapable conclusion is the following: If accelerated cosmic expansion is real and it seems that it is real, then Hawking's radiation, Rindler's horizon and Unruh's temperature are also real. In fact the applicability of Cosserat theory to Dirac's equation and the present work shows that we can regard spacetime as tangibly real and then everything will fall into place the right way. Around the year 2006 the Author pondered the question of which theory is more fundamental, relativity or quantum field [115, 116]. The question seemed at the time Goedelian undecidable. However with present understanding the Author tends to believe relativity is much stronger than we or even Einstein himself ever thought it is. This view seems to be correct when we look deeper at the present results connected to pure gravity and 't Hooft-Veltman renormalization [119, 120].

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To the memory of G.V.Kamenkov, outstanding mechanician, representative of Kazan Chetayev School of Mechanics and Stability

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Georgy Vladimirovich Kamenkov
12.01.1908 – 09.10.1966

Kamenkov Georgy Vladimirovich (12.01.1908 – 09.10.1966) is outstanding scientist in aerodynamics, theory of motion stability and nonlinear oscillations. He was born in January 12, 1908 (December 30, 1907) at a small station "Nochka" of Moscow-Kazan railway in the family of a railway worker. About his childhood and youth, little is known. At the age of seven, he went to study in real school, Saransk.

In 1924 Kamenkov's family moved to Kazan, where he continued his education. In 1926, eighteen-year-old Georgy went up to Kazan state University to study at the physics-mathematics faculty and in 1930 he had graduated from it.

In the same year he was awarded the qualification of a scientific worker of 2-d degree in mechanics and he became a postgraduate student of Kazan state University.

G.V.Kamenkov had received an excellent mathematical education in Kazan under the guidance of renowned scientists, mathe-maticians, professors N.N.Parfentiev, N.I.Porfiryev, N.G.Chebotarev, D.A.Goldhammer, D.N.Zeilyger, P.A.Shirokov.

Great abilities of a young postgraduate student for theoretical studies have been noticed and supported by his scientific advisor, an outstanding scientist, Professor N.G.Chetaev, Founder of Kazan School of Mechanics and Stability.

N.G.Chetaev highly appreciated the results obtained by G.V.Kamenkov in the years of post-graduate. In his comment concerning the work of postgraduate student, he wrote: "G.V.Kamenkov was the best postgraduate student at the Department of mechanics at Kazan University". His great mathematical ability was manifested in the current post-graduate studies and scientific research, and at the termination of postgraduate study he defended his PhD thesis on the theme: "On the vortex theory of head drag". Briefly his work was favorably described by Professor V.V.Golubev in proceedings "Mechanic for 15 years". In addition, at the end of the review on his work N.G.Chetaev wrote, "I warmly supported, on my behalf, the suggestion for G.V.Kamenkov to enroll him as a postgraduate student of the Academy of Sciences of the USSR, where under the guidance of Acad. S.A.Chaplygin he could usefully be progressed, as in hydro-aerodynamics as well as in analytical mechanics in a general sense".

These plans were not destined to come true, because after the defending of the candidate dissertation on a T.Karman vortex street in 1933 G.V.Kamenkov remained at work in Kazan.

On March 5, 1932, Kazan aviation Institute (KAI) was created and this year G.V.Kamenkov was enrolled to the staff of the teachers KAI; from the very beginning he took active part in creation and development of KAI, following in this to main principles of his Tutor-Teacher N.G.Chetayev. From 1933 he became an associate professor of the Department of Aerodynamics of aircraft, KAI.

In 1937 Georgy Vladimirovich had defended his doctoral dissertation on the topic "On the stability of motion", in the same year he became a Professor. His doctoral dissertation in 1938 was awarded the first prize of the Presidium of the USSR Academy of Sciences in the nomination of young scientists, he received the prize in the amount of 2000 rubles.

After defending his doctoral dissertation scientific activity had been continuously connected with organizational activity. From 1937 to 1944 Georgy Vladimirovich was the Deputy Director on scientific and educational work in KAI and in 1944-1949 – he is Director of KAI. During the war years G.V.Kamenkov at the same time was the head of the Kazan branch of the Central Aero-hydrodynamic Institute, had made much effort in the development of aviation technology. Due to his foresight and magnificent intuition of the mechanician-mathematician in КАИ in 1944 the new speciality for the country «Jet engines» was organized, with the foundation of the Deparment «Jet engines»in KAI on this speciality in 1945. This was the beginning of new area in training of engineers and research works in our Fatherland.

Since 1949 he was appointed Deputy Director on scientific work at Moscow Aviation Institute (MAI). At the personal request he was dismissed in 1955 (for health reasons). From 1956 up to 1958 he was the acting Director of the MAI, carrying out in MAI the N.G.Chetayev ideas about fundamental Higher Engineering Education in our country, including complex multidisciplinary area "aviation engineering". Since 1958 he became a head of the chair of Aircraft Aeromechanics, MAI. Under his leadership, 7 doctors of Sciences, more than twenty candidates of technical, physical-mathematical Sciences defended their dissertations. For many graduate students, staff of the Department, candidate for a degree from various industrial enterprises Georgy Vladimirovich was a reliable assistant who had helped to solve complicated applied and theoretical problems. The staff of the Department remembers him with great warmth and gratitude for his high spiritual qualities; they note that in his days he was simple, modest and accessible to any employee [1, 2].

G.V.Kamenkov was a Deputy of the Kazan city Council, Deputy of the Moscow city Council, was awarded the orders of Lenin, Red Banner of Labor, "Badge of honor" and medals. The Memorial plaque was installed on the aerodynamic building, MAI. Below we will give an overview of his scientific works with brief description of scientific results.

Almost all scientific works of G.V.Kamenkov are dedicated to fundamental problems of mechanics, mathematics, including the problems, formulated by A.M.Lyapunov, in particular, to aerodynamics, stability theory and nonlinear oscillations theory [3-17].

In his first work [3] in aerodynamics G.V.Kamenkov had investigated T.Karman vortex street stability. The author had shown that the conditions obtained by T.Karman along with N.E.Zhukovsky are wrong because they were derived from first approximation equations analysis only. He had shown that due to analysis of nonlinear equations the instability of vortex street had taken place.

Unsteady airplane wing motion was originally investigated in [4] (that was the continuation of S.A.Chaplygin research).

In his fundamental work [9] concerning wing theory in overcritical mode G.V.Kamenkov had got formulas defining drag force for any parameters values.
Another large part of G.V.Kamenkov works was dedicated to the study of critical cases of A.M.Lyapunov's stability theory of steady-state and periodic motion. The works [5, 6, 7] were the first after A.M.Lyapunov where more complicated critical cases of two zero and two pairs of pure imagine eigenvalues were investigated. The author had derived the necessary and sufficient conditions of stability defined by first nonlinear terms in very complicated equations of perturbed motion. This results essentially supplemented A.M.Lyapunov's classical investigations in the theory of critical cases. Later they were used by many other authors in the stability theory, control theory with applications to the problems of aviation technique.
In his monograph [8] G.V.Kamenkov had generalized the derived results and applied them to multidimensional systems. In particular general theorem on the instability of steady-state motion in the critical case of arbitrary number of zero eigenvalues together with the rest eigenvalues having negative real parts was postulated, as well as it was shown that the critical case of n pairs of pure imagine eigenvalues without any internal resonance may be reduced to the critical case of n zero eigenvalues with n groups of solutions. The possibility of reduction in this critical case is important when solving different applied problems in technique and nature where various critical cases of stability theory are occurred.
A great interest in stability theory was paid to the problem of stability of periodic motions in [12, 14]. In these works critical cases in which the stability conditions are defined by a set of nonlinear terms in equations of perturbed motion were considered. It was shown in [15] that using nonlinear transformations with periodic coefficients these cases can always be reduced to the critical cases for autonomous systems studied earlier.
As a rule while solving technical problems it's needed to know the behavior of the system not on the infinite time interval (this is formulated in A.M.Lyapunov stability theory) but on a finite (maybe very large) time interval. So it's necessary to introduce the definition of "stability on a finite time interval". Such a definition was given by G.V.Kamenkov in [10] along with the proof of a set of stability and instability theorems. Formulation of the problem of stability on a finite time interval and the method of its solving given by G.V.Kamenkov had occurred to be very useful in solving applied problems. This work [10] marked the beginning of a new scientific direction in the stability theory.
While developing the general stability theory in critical cases G.V.Kamenkov in [11] had come up to one of the most important and still not clear yet problem of study of stability in cases close to critical; the latter being very important in various applications especially in flight control problems. These cases are characterized by the existence of at least one eigenvalue with very small positive (or negative) real part among other roots of characteristic equation that corresponds to a system of first approximation (linearized system). Despite the fact that according to A.M.Lyapunov's definition of stability the problem here is solved completely by first order terms the permissible initial deviations may happen so small that in practical sense the conclusion about the stability of motion might be of no use. For such cases G.V.Kamenkov had given new practically important definition of stability. It required the existence of limited domain, from which no one current perturbation should leave if some conditions imposed upon nonlinear terms of the equations of perturbed motion were met, or when unstable under very small initial deviations, motion might be practically stable because maximum deviations do not exceeded allowable limits.
In this work G.V.Kamenkov has closely approached to a problem of nonlinear oscillations which was investigated in one of his last works [13]. Although the systems considered by the author contain a small parameter and so are quasi-linear, they essentially generalize previously known methods of the study of quasi-linear systems. The proposed new method

for finding nonlinear oscillations G.V.Kamenkov called “the method of Lyapunov functions”. In addition to theorems which give conditions for the existence of periodic solutions of multidimensional systems with a small parameter, the author had developed method of determining the admissible values of a small parameter, which guarantee the existence of periodic solutions. Autonomous and non-autonomous (periodic) system as well as systems that are not converting into linear when small parameter equals zero are considered. In this work criteria of stability of found periodic solutions were formulated.
The main research results of G.V.Kamenkov are given in the monographs [16, 17].

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Initiatives for Operational Research Education

Section at International Conference on Operational Research
(EURO-INFORMS 2013, Rome, Italy

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The purpose of the article is to focus on results of the projects aimed to support Operational Research education presented on the corresponded stream at EURO-INFORMS 2013 Conference in Rome, Italy.

Introduction

"Initiatives for OR Education" Stream was held on July 3, 2013 during EURO-INFORMS 2013 conference at Sapienza University of Rome, Italy (http://euro2013.org). The stream was mainly focused on exchanging experience about existing initiatives for OR education, with intention to share and to systematize common approaches and methods of creation, adoption and development of OR courses both in regular and extended education programs. The stream also gave a good opportunity to present and discuss existing and possible future initiatives for OR education.

Three sessions were organized and held within the stream:

1. "OR in Regular Study Programs"
2. "Additional Educational Activities for OR"
3. "OR Promotion among Academia, Businesses, Governments, etc.".

The Section participants presented the interesting scientific results in framework of Projects and researches from 7 countries.

Exchange of ideas for OR

OR in Regular Study Programs Session gathered scientists and practitioners interested in initiatives that exist within classical academic programs. Three presentations were given at the Session. Alberta Schettino (AIRO), was the first speaker. She shared the results of OR dissemination activities at Istituto Tecnico "G. Galilei", Italian secondary school in Imperia. This project aimed at young students with an intention to introduce them to OR and simultaneously improve their English listening skills [1]. The Project included:

1. a group on Facebook where an author could post a list of problems and various materials with a possibility to discuss in real time;
2. problems proposed and solved in class: 3 with optimal allocation of resources (profit maximization) and 3 product mixes (cost minimization);
3. resolution: a diagram where solution is calculated in all the vertices, and optimal graph is chosen by intuitive notion of gradient (Simplex method) using free version of Wolfram Alpha;
4. extensions introduction to the duality theory using a case of product mix problem and economic interpretation of duality (towards the concept of economic equilibrium).

Istituto Tecnico "G. Galilei" is planning to repeat and extend the project in 2014 with the same task in the 3rd form as well as to introduce transportation problems, CPM & PERT, and inventory management problem for the 4th form. Also they are going to hold "What is O.R."

seminar with at least one lesson about the Knapsack problem and a lesson about the Master Bay Plan problem.

Then, Fikret Korhan Turan from the Department of Industrial Engineering, Istanbul Kemerburgaz University, Turkey, presented his team's experience of implementing a set of investment projects that improve the University's sustainability performance. Survey results showed how stakeholders' priorities for a private university changed under low, medium and high financial constraints [2]. They used interactive software (www.superdecisions.com) for data collection, AHP tool for its analysis, and Weighted Geometric Mean Method to implement group decision making. Sustainability measurements include:

1. Teaching criteria (Average graduate salary, Faculty/student ratio, Number of students following graduate study, Student satisfaction);
2. Research & Development criteria (External research grants and awards, Number of graduate programs, Number of refereed publications, Qualification of graduate students);
3. Service & Social Responsibility criteria (Employee satisfaction, Environmental footprint, Local community collaborations);
4. Financial criteria (Revenues / expenses ratio).

The researchers used AHP/ANP as group decision support tools for participatory management in higher education for organizational sustainability. Stakeholders' preferences were analyzed. The team discovered the following:

- Similar pattern existed in priorities for different stakeholders, indicating harmony;
- Stakeholders agreed on the importance of HR & Intl and Labs & Library projects;
- Stakeholders appear to think that projects' contributions vary according to performance dimensions considered;
- Stakeholders' preferences change under financial constraint;
- Projects with "high visibility" gain importance as the level of financial constraint increases.

In their further research they are planning to: extend the study increasing number of participating stakeholders up to 40-50; consider both internal and external stakeholders; collect data through the Internet (online application of ANP); use web-ANP solver (http://kkiry.simor.mech.ntua.gr/Rokou/ANPWEB/); conduct a similar study at a different university; statistically analyze the obtained results ANOVA, ratio test, distribution free tests; plan investments using results; develop an optimal investment portfolio balancing satisfaction of stakeholders; consider timing and uncertainty issues (dynamic AHP/ANP and sensitivity analysis).

Jo Smedley from Centre for Excellence in Learning and Teaching at the University of Wales (Newport UK) showed how OR soft systems approaches are used in education. Published results prove that quality of learning abilities using OR soft systems approaches got evidently improved [3].

During Additional educational activities for OR Session, participants presented their projects aimed at spreading and improving knowledge about OR. Four abstracts were detailed within the session. Experience of teaching a blind student Linear Programming using Excel-Solver was discussed by Laura Plazola Zamora from Mexico [4]. She was teaching a student to solve Linear Programming problem, where decision variables, objective function and constraints set were built from plastic letters (from A to Z), numbers(from 0 to 9), and also mathematical operators and symbols.

Project had the following difficulties:

- the student forgot how he had allocated variables;
- use of space made it difficult for him to work;

- searching and formulation time was considerable;
- letters and symbols were insufficient.

The student suggested using Excel to set the problem. Usage of MS Excel decreased time consumption and increased efficiency of work. They built a Cartesian plane in a cork board with thumbtacks. They used material with different textures. When touching, he could identify areas formed by the constraints. The feasible area was identified and represented with a piece of rough paper. The student confirmed that the use of different materials allowed him to clearly understand the areas formed by the constraints. He found it hard to modify numeric scales of the axes. Material properties require vertices of the feasible area to be within an integer coordinate. In addition, it was hard to use too many restrictions.

Finally, student took a test, where he had to demonstrate what he had learned. His result was 4 right answers out of five questions. Comparing his results with those of his 9 schoolmates, considering the 5 linear programming reactants in the test, we had the following results: 3 students had 0 right answers; 4 students - 1 right answer; 1 student - 2 right answers, and 1 student - 4 right answers. Conclusion is that it is true that the concepts of Geometry do not come readily to a blind person, because of its spatial content. There is no reason for a person with sufficient abilities to fail to become a successful mathematician, engineer or have a degree in human resources simply because he or she is blind.

Olga Nazarenko from National Technical University of Ukraine (Kyiv, Ukraine,)described methodology for developing and approbation of academic courses through usage of additional educational activities based on case study of Summer School AACIMP (http://summerschool.ssa.org.ua/) [5]. Summer school is held annually in Kyiv, Ukraine since 2006. 39 tutors and 76 attendants participated in Summer School in 2012. In 2013 it consisted of 4 parallel streams:

1. Neuroscience - gives you a great opportunity to understand one of the most fundamental questions in the modern science - how the brain works. Lectures will introduce you to basic principles and techniques for analyzing, modelling, and understanding behaviour of cells and circuits in brain. Participants of the stream are not expected to have any prior knowledge in the field.
2. Operational Research - introduces methods of mathematical analysis to facilitate solving complex challenges from real life of business and government. This year it will mainly focus on open questions in financial mathematics and supply chain management.
3. Applied Computer Science - provides a broad introduction to machine learning and artificial intelligence areas. Lectures will be based on real-life case studies and applications. For example, you will get to know how to build an algorithm that plays chess better than Harry Kasparov (search algorithms and heuristics), or protects your e-mail from spam (text recognition), or makes a robot search for the specified object in the room, or become useful in many other applications.

The project enables new learning possibilities and interdisciplinary communication, facilitates sharing ideas among scientists.

Giuseppe Bruno from Napoli (Italy) and Andrea Genovese from Sheffield (UK) introduced a Summer School project for training a specialist in optimization and decision support systems for supply chains management that is held within Erasmus Intensive Programme (http://w2.estgp.pt/docentes/jlmiran/Odss.SC2013/) [6]. 28 MSc Students and 8 PhD students, lecturers, assistant researchers from Italy, Portugal, India, China, Germany, Jordan, Mexico, Netherlands, Spain, South Africa, Sweden, Taiwan, and Ukraine participated in the project in 2013. A truly interdisciplinary team has been brought together, including the following areas:

§ Information Systems Management

§ Operations, Logistics and Supply Chain Management
§ Operational Research/Management Science
§ Chemical Engineering
§ Accounting and Business Studies

The Programme included visiting industrial partners: Jeronimo Martins – food distribution, Valnor – urban waste collection and recycling, Evertis – plastic packaging from recycled plastics, Delta – coffee manufacturing.

Results of the Programme are inclusion into the “Best Practices” handbook from the Portuguese LLP agency, successful re-application (including University of Sheffield as an official partner). Authors discussed a case of International Cooperation in OR Education, aiming to bring together approaches and expertise into different European higher education institutions. Odss.4SC summer school addresses general aspects of SCM, with a special emphasis on sustainability, optimization procedures, and decision support systems.

Andrea Aparo from Sapienza, University of Rome (Italy) and Marco Fida from Politecnico di Milano, Genoa, (Italy) presented a framework of DISD Master Programme that aims at managing analytical methods and stochastically behaving persons in an integrated way [7].

Within OR Promotion among Academia, Businesses, Governments, etc. Session, Wen Ju Ko from National University of Kaohsiung (Taiwan) [8] revealed and explained results of science and technology policy evolution in Taiwan .

Berk Orbay from Bogazici University (Istanbul, Turkey) explained usage of an adaptive curriculum algorithm for Turkish high school education system [9]: completely automated, provides granular analysis, supports distance learning only, allows student intervention. The first version was released in April 2013 supporting Math only with no optimization models contain 4,000 participants. Each topic in math requires a number of study hours and weight. Weight depends on the role of a topic in the math exam. First the students were asked how many hours he/she will study math per week (10/20/30 hours). Then, correct/wrong answers in tests are counted for each topic. This results in performance percentage (%). If no feedback is given, performance is 50% for each topic. A schedule is given to a student for the next week. Decision rule is simply improvement in performance over number of hours studied (linear learning function).

The first version of the programme stayed online till mid-June 2013. It featured mixed blessings, high drop-out rates, too much reliance on student feedback, no test questions for self assessment, positive feedback and strong demand for other courses (i.e. chemistry, physics).

The second version (September 2013) had all courses included, testing within the system, careful difficulty adjustments, improved learning functions, separation of learning and training, inclusion of speed criterion, performance prediction, was more in line with Item Response Theory*.

One can note that adaptive learning systems are becoming increasingly prominent. Students are open-minded, even keen about it. It has potential to improve learning experience and bring equality into classroom. It is certainly an optimization problem with all its aspects. ParlakBirGel preliminary analyses show that there is strong demand for it in Turkey. The researchers plan to measure students’ performance in terms of convergence, variance and speed.

Vassilis Kostoglou from the Department of Informatics, Alexander TEI of Thessaloniki, Greece, suggested a tool to improve vocational orientation and employability of students and young graduates: developing professions’ digital guide [10]. This tool has Job Profile Library, main characteristics of which are:

1. Thousands of jobs with their essential duties.

2. Context-sensitive advice is available whenever you need it.
3. Over 30 key performance competencies with multiple factors.
4. Interview question and form generator creates a list of custom interview questions and rating form for comparing candidates
5. Built-in word processor enables merging duties from multiple jobs and edit, print, export, or even import existing job description

Authors are going to embed historical data into the database helping to answer additional more complex queries as well as data related to graduates placement in the labor market.
Martin Kunc from Warwick Business School, University of Warwick (UK) discussed impact of OR communities on economics and social development in Latin America countries.

Conclusion

Initiatives for OR Education Stream provided its participants with a platform to present and discuss planned or existing EURO and IFORS initiatives for OR education and projects aimed to promote OR among academia, business, governments, etc. This Stream was a successful event facilitating professional development of OR community.
We cordially thank the two Co-Chairs of the Programme Committee at EURO-INFORMS 2013: Prof. Dr. Marc Sevaux and Prof. Dr. David Simchi-Levy, the Chair of the Organizing Committee, Prof. Dr. Paolo Dell'Olmo, as well as PC Member, responsible also for the Main Area "OR Education, History, Ethics", Prof. Dr. Maria Antónia Carravilla, and EURO Vice President Prof. Dr. José Fernando Oliveira for their interest, recommendations, and continuous support.

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Problems of continuum mechanics

Scientific Seminar and Final Scientific Conference'2013
(Kazan, 2013-2014)

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The Scientific Seminar "Problems of Continuum Mechanics" works in the Institute of Mechanics and Engineering, Kazan Science Center RAS. The Chairman of the Seminar is Director of the Institute Corresponding Member of the RAS D.A.Gubaidullin. Problem reports and dissertations of researchers of the Institute and scientists from other organizations are presented and discussed at the Seminar.

The reports, representing the achievements in the sphere of non-linear mechanics of thin-walled constructions, hydroaeroelastic and wave systems; dynamics of multiphase multicomponent media in porous structures and technological installations; non-linear stability theory of control systems with changeable structure, were shown on the section of the Final scientific conference-2013.

The Ph.D theses in mechanics of liquid, gas and plasma and mechanics of deformable solid body, were presented at the Seminar.

The abstracts of the reports are presented below.

The reports at the Final conference

February 10, 2014.

D.A.Gubaidullin, R.G.Zaripov, L.A.Tkachenko (IME KazSc RAS).
Oscillations of the aerosol in a half-open pipe with flanges near resonance frequencies in the shock-free mode.

The nonlinear oscillations of small-disperse aerosol are experimentally studied in a tube with a flange in the shock-free mode near the resonance frequency. The dependences of oscillation swing of pressure of aerosol on the frequency are received at different inside diameter of the flange. Dependences of number concentration of droplets of oscillating aerosol from time are given. The effect of frequency and amplitude displacement of the piston and the inner diameter of the flange on the time scale of the clearing aerosol is studied. It is shown that the process of clearing of the oscillating aerosol is 5-10 times more effectively than natural precipitation.

D.A.Gubaidullin, A.A.Nikiforov (IME KazSc RAS).
Calculation of the acoustic signal distortion in the diagnosis of multilayer samples with bubble inclusions.

The dynamics of pulse pressure disturbances in a liquid containing a multilayer wall is investigated theoretically. With use of fast Fourier transform algorithms the distortion of the acoustic signal for the diagnosis of a multilayer sample, comprising a layer of liquid with the polydispersed bubbles is calculated. A good agreement between theoretical and experimental data is obtained.

D.A.Gubajdullin, A.A.Nikiforov, R.N.Gafiyatov (IME KazSC RAS).
Acoustic waves in multifraction bubble liquids.

The propagation of the acoustic waves in multifraction mixtures of liquid with vapor–gas and gas bubbles of different sizes and compositions with phase transformations has been studied. A system of the differential equations of the motion of the mixture is presented, and the dispersion relation is deduced. The evolution of the weak pulsed perturbations of the pressure in this mixture is calculated numerically. It is shown that the dispersion and dissipation of the acoustic waves is significantly affected by different fractions of bubbles in the disperse phase.

D.A.Gubaidullin, Yu.V.Fedorov (IME KazSC RAS).
Acoustic waves in two-fraction liquids with polydispersed gas-vapor bubbles.

The propagation of acoustic waves in two-fraction liquids with polydispersed gas-vapor bubbles in the presence of phase transitions is studied. A mathematical model is proposed, the dispersion relation is obtained. Comparison of the theory with experimental data is presented.

D.A.Gubaidullin, P.P.Osipov, A.N.Zakirov (IME KazSc RAS).
Influence of basset force on the direction of inclusions drift in a standing wave.

The inclusion drift in a standing sinusoidal fluid-velocity wave at various Reynolds and Strouhal numbers under the action of the viscous force, the virtual mass force, the buoyancy force and Basset force has been investigated. For a given inclusion density, as the standing wave frequency increases, its threshold value, above which the direction of the wave force reverses, is attained sooner or later. For various Reynolds and Strouhal numbers (with and without Basset force), the dependences of the squared threshold drag coefficient on the inclusion density number have been found. These dependences show that with increasing Reynolds and Strouhal numbers the threshold value of the squared drag coefficient decreases markedly. Basset force influence on a threshold value has been investigated. It has been shown, that Basset force influence most on threshold values of low-density inclusions. When the Strouhal number is increased the threshold value of the inclusion density is decreased.

A.L.Tukmakov (IME KazSc RAS).
Thermoacoustic oscillations of a gas mixture of electric-charge CO_2 laser.

The paper describes the dynamics of the gas mixture in cylindrical resonator of CO_2 laser with periodic allocation of heat in the vicinity of its axis. The dynamics of the gas is described on the basis of a numerical solution of the system of Navier-Stokes equations by explicit MacCormack’s method with splitting of the original operator on the space directions. The fields and ranges of change of the gas-dynamic functions are defined and the average flow of the gas mixture is built.

D.A.Tukmakov (IME KazSc RAS).
The Riemann problem for the two components gas.

On the base of explicit finite deference method of MacCormack the system of dynamics of two component viscous compressible heat conductive gases has been solved. In this work numerical solution of problem of chock tube has been compared with analytical solution which is known from literature.

D.A.Tukmakov (IME KazSc RAS).
The way to increase the intensity of gas fluctuation generated in a tube

In this work the approach of increase of intensity of gas fluctuation in acoustic resonator by the alternative construction of piston with invariable power and invariable amplitude of fluctuation of executive element is presented. Numerical calculations for different configuration of piston mean have been made.

R.I.Baianov, A.L.Tukmakov (IME KazSc RAS).
A numerical model of a single-speed vapour-gas-liquid environment in diffusion approximation.

The mathematical model and numerical method for the description of the dynamics of vapour-gas-liquid system in diffusion approximation based on the decision of the Navier-Stokes equations for compressible heat-conducting gas by the explicit Mac-Cormack's method are presented. The verification of the numerical scheme by comparison with data of the experiment and with analytical solutions of the problem in a one-dimensional setting is made. The stationary fields of pressure, speed, temperature and partial densities of vapour-gas-liquid mixture components have been obtained for the case of flow around the cylinder taking into account the phase transitions.

A.A.Aganin, A.I.Davletshin, D.Yu.Toporkov (IME KazSc RAS).
Simulation of a strong enlargement-compression of cavitation bubbles in a comet-like streamer.

A mathematical model of dynamics of weakly nonspherical bubbles located in a line (as a streamer) during their enlargement-compression is proposed. In the model, the enlargement-compression is divided into two stages: the low-speed (the liquid is weakly compressible, the vapor in the bubble being ideal homobaric) and high-speed (motion of central bubbles is described by equations of gas dynamics for the spherical component of the liquid and vapor motion) ones.

A.A.Aganin, A.I.Davletshin (IME KazSc RAS).
Transform of spherical functions for spatial modeling of interaction of bubbles in liquid.

A compact expression of transform of spherical functions has been derived in the case of transition from the system of co-ordinates with the origin at the center of one bubble to that with the origin at the center of another. Equations of spatial interaction of spherical bubbles with a third order of accuracy relative to the small parameter, the ratio of the sum of radii of the bubbles to the distance between their centers, have been derived with using this expression.

T.F.Khalitova (IME KazSc RAS).
Deformation of the shock waves in the bubble depending on the amplitude of its initial non-sphericity.

Dependence of the shock wave distortions inside the spheroidal cavitation bubble in the acetone on the amplitude of its initial non-sphericity is investigated. It is shown that in the final stage of compression of bubbles with moderate non-sphericity shock wave focusing is similar to the impact of the two planar waves when the bubble is prolate and is similar to the cylindrical shock wave focusing when the bubble is flattened.

A.A.Aganin, L.A.Коsolapova, V.G.Malakhov (IME KazSC RAS).
Dynamics of a cavitation bubble in a liquid near a rigid wall.

Dynamics of axisymmetric cavitation bubble, variation of the surrounding fluid pressure and velocity fields during its collapse near a plane rigid wall is considered. The bubble at the beginning of its collapse is spheroidal. The collapse is examined until collision of any parts of the bubble surface. The fluid is assumed inviscid and incompressible. Deformation and displacement of the bubble surface and the change of the fluid velocity is computed by Euler's scheme using the boundary element method. Influence of initial deviation of the bubble shape from the spherical one and fluid properties is investigated. The range of values of the ratio of semi-principal axes, for which a cumulative jet directed perpendicularly to the wall is formed on the cavity surface, is investigated for vapor bubbles in water and acetone.

A.A.Aganin, N.A.Khismatullina (IME KazSc RAS).
Impact on elastic-plastic body of a liquid jet arising during collapse of a cavitation bubble.

A technique of numerical investigation of elastic-plastic deformations in a body under jet impact to its surface is developed. A number of features of dynamics of near-surface part of the body, location of the yielding zones and variation of their configuration, effect of load non-uniformity arising under the jet impact has been studied depending on the jet velocity.

M.S. Ganeeva, V.E. Moiseeva, Z.V. Skvortsova (IME KazSc RAS).
Buckling safety membranes under the action of liquid pressure and temperature.

Nonlinear bending and stability of buckling membranes appearing as spherical segments with their convex side exposed to the pressure of the heated or cooled liquid (as in the working environment of the potentially explosive device) were studied. The results of numerical calculations depending on environment temperature level, material characteristics and reduction of segments thickness in the pole area were obtained.

A.I.Malikov (IME KazSc RAS).
Finite-time stability and boundedness of hybrid switching systems.

The stability and boundedness conditions of hybrid systems with switchings on a finite time interval are presented. Uncertain perturbations, the nonlinearity, satisfying to conic conditions, various assumptions about modes switching functions are taken into account. The ways of state estimation and transient performance for considered systems are given on the base of matrix comparison systems

<u>February 11, 2014.</u>

B.A.Snigerev (IME KazSc RAS).
Flow of polymer melt in complex channel.

The flow of polymer melt in channel of complex form is studied. The fluid motion is described by equations of conservation of mass, momentum with multi-mode rheological constitutive equation of reptation type model. This model developed from kinetic theory of polymers. The considerable influence of fluid stress relaxation time on the size and intensity of vortex zone in front of contraction in channel is numerically shown.

I.V. Morenko, V.L. Fedyaev (IME KazSc RAS).
Hydrodynamics and heat transfer of the rotating circular cylinder in the suspension flow.

The nonisothermal unsteady viscous flow with an impurity past of the rotating circular cylinder was studied. Mathematical modeling of these processes is based on the Lagrangian approach taking into account the mechanical and thermal interaction between the carrier and dispersed phases. The influence of the impurities on the flow was analyzed. The drag, amplitudes of the lift coefficient, Nusselt number depending on the relative rotational speed of the cylinder was established.

M.Kh. Khairullin (IME KazSc RAS), **R.G.Farhyllin** (ASPI), **E.R.Badertdinova** (KSTU), **V.R.Gadilhina** (IME KazSc RAS).
Thermohydrodynamic investigations of vertical wells operating in multilayer reservoirs.

A computational algorithms based on the theory of ill-posed problems are proposed for estimation of filtration parameters of multilayer reservoirs using the result of thermohydrodynamic investigations of vertical wells.

P.E. Morozov (IME KazSc RAS).
Interpretation of inflow curves of horizontal wells.

Analytical and semi-analytic solution of unsteady fluid inflow into the horizontal well after an instantaneous sampling or injection is proposed. The effects of horizontal well length, wellbore storage coefficient and permeability anisotropy on the pressure and inflow rate in horizontal well are studied.

A.I. Abdullin, M.N.Shamsiev (IME KazSc RAS).
Investigation of thermo-hydrodynamic processes in the system "reservoir – multi-wellbore horizontal well".

In this work a three-dimensional mathematical model of heat and mass transfer in the system “reservoir – multi-wellbore horizontal well” is proposed. It’s used for the prediction of temperature and pressure profiles after well startup. A computational algorithm for the interpretation of thermo-hydrodynamic studies of multi-wellbore horizontal wells is proposed.

A.I. Nikiforov, R.V. Sadovnikov (IME KazSc RAS).
Wave impact on a porous medium saturated with two immiscible liquids

The propagation of plane waves in a porous medium saturated by two immiscible liquids with the capillary pressure difference between phases was considered. A dispersion relations were obtained for the three phase velocities of longitudinal and transverse waves in the system of a porous skeleton, wetting and non-wetting liquids. The impact of low frequency vibrations in the range of 1-50 Hz on the phase velocity was investigated for different ratios of viscosity of wetting and non-wetting

T.R. Zakirov, A.I. Nikiforov (IME KazSC RAS).
Analysis of changes in well productivity resulting from the acid treatment of three-dimensional models based on two-phase flow.

The problem of acid-treated bottom zones of injection and production wells is considered. It is shown that as a result of realized events it was managed to increase the oil recovery rate by 1.6%. Using parallelization technology OpenMP it was able to reduce the computation time by 2.7 times.

A.V. Elesin, A.Sh. Kadyirova (IME KazSc RAS).
Influence of the measurements time of the well debit on the results of the absolute permeability identification.

The absolute permeability identification on the well debit measurements is considered in reservoir with different by structure heterogeneity. The influence of number and time of measurements on the identification results is explored. It is shown that the using a priori comparative information about the identified values improves the results of identification.

A.V. Tsepaev(IME KazSc RAS).
The methods of solution of the three-phase flow equations with gravitational and capillary forces.

The new numerical algorithms based on domain decomposition methods have been developed for solving the three-phase flow equations (water, oil, gas) in porous media with mass interchange between the oil and gas phases. To calculate the mass interchange between the oil and gas phases the solubility coefficient is used. The gravitational and capillary forces are taken into account. These algorithms have been realized on new generation heterogeneous computing system, built using modern central processing units and graphics accelerators.

G.A. Nikiforov (IME KazSc RAS).
Numerical solution of two-phase flow with a nonlinear law of motion.

Results of numerical modeling of two-phase flow of non-Newtonian fluids in porous media are presented. The problem is solved by the method of control volumes in variables "velocity-saturation." On the example of a five-pointed water flood the calculations for various non-linear laws of motion were presented. Comparison of simulation results showed that the residual oil and oil recovery may be very difference depending on the law of motion for the same initial and boundary conditions. Significant differences were observed in the form and saturation of stagnant zones.

<u>February 20, 2014.</u>

N.M. Yakupov (IME KazSc RAS).
Fragment from history of development of a science.

Depending on used materials, energy, level of available technology, application of a mathematical apparatus, etc. five stages of a birth and science development are allocated. The attention is focused on first two stages of development - stages of development of the Ancient world, including the periods of Ancient Rome and Byzantium, and also the Islamic period of the development, which laid the foundation for a modern science. Sources: the Science, The publishing house Readers Digest, 2012, 512 p.; the Discoveries which have turned the world. As it was. Publishing Kontent, 2008. 224 p.; the World history. People, events, dates. The publishing house Readers Digest, 2007, 576 p.; The Atlas of travel. Изд. The publishing house Readers Digest, 2012, 287 p.

N.K. Galimov (IME KazSc RAS).
Definition of stress on a surface of the bent round plate.

For a research of influence of deformation on corrosion in two-axial intense samples, the experimental device (the patent №2437077) has been developed. The deformed round plate is a part of this device. For an estimation of level of stress on a surface of such sample the problem of a bend of a round plate from the loading enclosed on a concentric circle has been solved. Thus the plate is leaned on a ring line with smaller diameter. The nonlinear problem is solved by Ritz method. Reliability of the received decisions is shown.

S.N. Yakupov (IME KazSc RAS).
Analys of system «a substrate - a covering».

Works on definitions of mechanical characteristics of coverings in system "covering-substrate" are continued. When the properties of a substrate of a polymeric film and a package «a film - the titan oxyde » were separately investigated, it was established, that the covering presence affects on the deflections of samples, thus with increase in a thickness of a covering the effect increases.
The approach to define the adhesive properties of films to a substrate is developed taking into consideration height of a covering and diameter of the basis of the formed dome. It is established, that the total size of parameter of a separation is practically invariable. The offered way is recommended for thin flexible coverings.

A.A. Abdiushev (IME KazSc RAS).
About analysis of SSB (stress strain behavior) of supported thin-walled structures.

To analyze the strength of reinforced shells by FEM in terms of displacement a technique to obtain shear finite element sheathing panels with ribs which are equilibrium by efforts (Ebner – Belyaevs model), based on the complete system of equations of structural mechanics was developed. When the shear of panels as the plane stress objects in the absence of edges was determined the degeneration of equilibrium elements in to hybrid finite elements with constant shift was indicated. The energy distance between the hybrid and the equilibrium models was quantitatively separated.

R.R. Giniyatullin (IME KazSc RAS).
The method of an experimental research of a corrosion of thin-walled samples.

Among the most widespread ways of an estimation of a corrosion deterioration are: gravimetrical way which is connected with the control of loss of weight of metal and an electrochemical way, connected with measurement of speed of corrosion on the basis of polarising measurements. These ways do not allow to define the degree of change of rigidness characteristics of thin-walled elements at corrosion. In this connection in the laboratory of NMS IME KazSC of the Russian Academy of Sciences the two-dimensional experimentally-theoretical method of the definition of integrated characteristics of thin-walled elements of the designs is developed, allowing to define the changes of rigidity of the investigated sample.

N.M. Yakupov, H.G. Kijamov, R.R. Giniyatullin (IME KazSc RAS).
An estimation of pressure in the thin deformed samples at a research of corrosion deterioration.

The basic moments of a research of corrosion deterioration under the stress are marked. The scheme of the experimental device and a research technique is described. For the bent plate in the considered device distribution of the stress is defined, using spline variant of a method of final elements with use of three-dimensional elements. Results of corrosion deterioration of the deformed plates are resulted. It is noticed, that the higher the level of stretching stress, the higher degree of corrosion deterioration in these areas.

N.M. Yakupov, L.U. Sultanov (IME KazSc RAS).
The analysis of concentration of pressure in toroidal covers with defects.

In the service on surfaces of covers there can be defects (local deepenings, cracks, slots, scratches and others) which can become a source of rupture of all design. By the digitization of thin-walled designs with coverly elements it is impossible to define concentration of stress of not through defects on a thickness. The analysis of a toroidal cover with not through local defect is made. For definition of intense - deformed condition of fragments of toroidal covers the educational variant of calculation complex ANSYS was used. It is noticed, that the maximum stress at defect deepening are displaced on an external surface.

S.N. Yakupov, L.U. Kharislamova (IME KazSc RAS).
The method of an estimation of durability of biological membranes.

Researches in the field of bionics show perfection of structures of elements of biological designs. Natural creations are the sample of an optimality of designs. The review of the works devoted to research of mechanical characteristics of biological membranes is resulted. Known approaches of an experimental research of properties of noted membranes are marked. The two-dimensional experimentally theoretical approach of research of mechanical properties of biological membranes, and also the device, allowing to realise the offered two-dimensional approach are described. The concrete results of experimentally - theoretical research of a peel apples are resulted.

N.M.Yakupov, H.G.Kijamov (IME KazSc RAS), **K.A.Kolyadov, A.R.Nizamov** (KSACU)
To calculation of rod systems.

At calculation of rod systems known approaches are used. It is supposed, that under tension stress, which appears in section, distributed in regular intervals and under bend stress allocation is linearly. On a frame example it is shown, that in areas of jamming, at the junction of the frame elements, in places where loads applied and where is local support, stress redistributed and may exceed the stress meanings received on the standard methods. For definition of the VAT of frame two-dimensional and three-dimensional finite elements on basis of complex LIRA and spline variant of finite element method were used. A conclusion: in critical areas it is necessary to perform additional calculations using two-dimensional and three-dimensional elements.

N.M. Yakupov, G.G. Gumarov (IME KazSc RAS).
Effect of mechanical strain on the changes in the physical properties of the surface of thin samples.

It is known that the deformation field promotes earlier destruction of passivating layer occurring on the metal surface from corrosion. To elucidate the mechanism of influence of deformation on the corrosion has been hypothesized: "Mechanical deformation influence on the electrochemical corrosion process by the change of the magnetic characteristics." Studies of the magnetic characteristics of thin films with ion-synthesized silicides of iron showed that the distribution of magnetic properties (coercive force distribution and the anisotropy field) depends on the stress level. Work in this direction will continue.

Reports at the Seminar

June 19, 2013
Y.A. Yumagulova (Bashkir State University, Birsk Branch).
Dynamics of pressure in confined spaces due to phase transitions.

On materials of the thesis submitted for Candidate of Sciences Degree, specialty 01.02.05 – mechanics of liquid, gas and plasma. Scientific adviser: Dr Sc, Professor, Academician of the Academy of Sciences of RB V.Sh. Shagapov. Reviewer: Dr Sc, Prof. A.A. Aganin.
The paper presents the results of a theoretical study of the process of reduction of the vapor pressure located in a closed volume due to condensation at the interface with the cold liquid on a horizontal surface, solid, and under the injection of droplets. It is shown that for the elimination of the consequences of accidents of power equipment caused by high pressure steam in a closed volume, the most appropriate to use the injection of steam drops in temperature by adjusting the volume of water and its contents.

July 5, 2013
R.R. Giniyatullin (IME KazSC RAS).
Research of changes of mechanical characteristics of the metal thin-walled elements which are in excited environments and at influence of physical fields.

On materials of the thesis submitted for Candidate of Sciences Degree, specialty 01.02.04 – mechanics of a deformable firm body. Scientific adviser: Dr Sc, Professor N.M.Yakupov. Reviewer: PhD N.K.Galimov.
In the work a definition of influence of mechanical deformations of a surface, magnetic field, ultra-violet radiation, the updating of a surface by a method of ionic implantation on rigid characteristics of the thin-walled elements which are in the corrosion environment is presented using two-dimensional experimentally-theoretical method.

November 19, 2013
V.V. Koledin (Bashkir State University, Birsk Branch).
The development of the instability of steam, gas and steam bubbles in a superheated liquid.

On materials of the thesis submitted for Candidate of Sciences Degree, specialty 01.02.05 – mechanics of liquid, gas and plasma. Scientific adviser: Dr Sc, Professor, Academician of the Academy of Sciences of RB V. Sh. Shagapov. Reviewer: A.A. Nikiforov.
The present work is devoted to the analytical and numerical study of the growth of a single vapor bubble in an infinite homogeneous volume of superheated liquid , as well as the

generalization of the results to the boiling liquid with many bubbles on the basis of the standard "cellular " model. Despite the significant history of the study of growth of vapor bubbles in superheated liquids , this research continues to attract the attention of researchers and has not lost its relevance in view of the importance applications associated primarily with nuclear energy.

Potential participants are kindly invited for the presentation of their results at the Seminar.
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Damir A.Gubaidullin, the Director of IME KazSC RAS; the Deputy Chairman for research of Kazan Science Center RAS; the Corresponding Member of the RAS; the specialist in the area of mechanics and thermo-physics of multiphase media.
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