Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 6, pp. 47–54, 2001 *UDC 517.957*

REDUCIBILITY OF NONLINEAR DIFFERENTIAL EQUATIONS TO BLOCK-TRIANGULAR FORM

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1. In solving the stability problem by applying the first Lyapunov method, an important role is played by the notion of reducibility of linear homogeneous differential equations (see [1], p. 43; [2], p. 154). The equations to be reduced possess rather large similarity in the qualitative behavior of solution, in particular, the property of stability is preserved as well as characteristic exponents of solutions (see [2], p. 154). To investigate the properties of a concrete equation it is important to establish its reducibility to an equation whose properties can be subject to study. In Lyapunov's works, in the capacity of such simplest equations those with constant matrices were considered. As is known, not any linear homogeneous differential equation can be reduced to an equation with constant matrix. However, Perron proved that any linear homogeneous differential equation can be reduced to an equation with triangular matrix, which then can be integrated. In [3] (p. 266) and [4] a criterion was obtained for reducibility of a linear homogeneous differential equation to an equation with a block-triangular matrix, which generalizes the result obtained by Perron.

In [5]–[7] the notion of reducibility on the set Ξ of nonlinear differential equations

$$\frac{dx}{dt} = f_0(t,x) \tag{1}$$

was introduced, where $t \in [T,+\infty)$, $x \in R^n$, $x = \mathrm{colon}(x_1,x_2,\dots,x_n)$, $f_0 \in C([T,+\infty) \times R^n, R^n)$, $f(t,0) \equiv 0$, all solutions $x(t : t_0,x_0)$ exist on the whole half-segment $[T,+\infty)$ and are uniquely determined by the initial data (t_0,x_0). Let a group of transformations of the set Ξ be given, whose invariants are characteristic exponents of solutions and the stability of zero solutions. Then this group is said to be Lyapunov, transformations which enter into this groups are called Lyapunov transformations, and the differential equations related to these transformations are said to be reducible (see [7]).

One of most frequently used transformation groups is the group LG [6], consisting of transformations $L \in C^1([T,+\infty) \times R^n, R^n)$, possessing the following properties:

1) $L(t,0) = 0$;

2) $\|\frac{\partial L(t,x)}{\partial x}\| \le K_L$ for $t \in [T+\infty)$, $x \in R^n$, K_L does not depend on t, x.

Here and in what follows we use the Euclidean norm for vectors and matrices $\|(a_{ij})\| = \sqrt{\sum\limits_{i,j} a_{ij}^2}$, $\|\mathrm{colon}(x_1,x_2,\dots,x_n)\| = \sqrt{\sum\limits_i x_i^2}$.

All the results obtained in this article are related to the reducibility in the group LG.

The use of the reducibility of nonlinear differential equations allows us to apply the ideology and technique of the first Lyapunov method to investigation of solutions of essentially nonlinear differential equations. As in the linear case, it is important to establish whether the differential equation under consideration is reducible to an equation of a simpler form [5]–[8]. This article