Characters and Coverings of Compact Groups

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Abstract—We consider characters and finite-sheeted coverings of compact connected Abelian groups and prove analytic and algebraic properties of characters. As an application of these results, we show that the character group of a compact connected Abelian group with trivial finite-sheeted coverings is divisible.

INTRODUCTION

This paper is devoted to the characters and the finite-sheeted covering mappings of compact connected Abelian groups. We study properties of the characters and functional equations with a view towards considering the structure of the covering mappings.

The motivation for our work comes from the following results and papers.

First, it is E. R. van Kampen's theorem on a factorization of the invertible elements in the Banach algebra of all continuous complex-valued functions defined on a compact connected group [1] (see Section 1).

Second, these are the papers [2] and [3]. In [2], the problem on the complete solvability of the polynomial equations with continuous coefficients is studied. In particular, it was shown that the solvability of the equations is closely related to the possibility of division in the first Čech cohomology group with integer coefficients ([2], §§ 1, 3). Note that the complete solvability of an equation without multiple roots is equivalent to the associated polynomial covering being trivial (see the definitions in Section 1). In [3], the main tool for studying covering mappings from topological spaces onto topological groups is the existence of a topological group structure on a covering space such that a given covering mapping endowed with this structure turns into a morphism between topological groups.

The following criterion is valid ([2], theorem 1.8 and its corollaries; [3], theorem 3).

Criterion for coverings triviality. *All n-fold covering mappings onto a compact connected Abelian group are trivial if and only if its character group is $n!$-divisible.*

The paper is organized as follows. It is divided into three Sections.

Section 1 contains definitions, notations and results from the theories of topological groups and covering mappings which will be used throughout the paper.

In Section 2, we prove an analytic property of the characters concerning their representation as the composition of the exponential function and a continuous function (Theorem 1).

Algebraic properties of the characters are studied in Section 3. To prove Theorem 2 about the roots of functional equations we make use of E. R. van Kampen's theorem mentioned above and Theorem 1. These results allow us to prove the necessity part of criterion for coverings triviality (Theorem 3).

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