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CONSTRUCTION OF THE INTEGRAL REPRESENTATION FOR SOLUTION OF EQUILIBRIUM EQUATIONS IN CERTAIN VERSIONS OF THEORY OF SHELLS WITH A COMPLICATED GEOMETRY

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In the present article we consider isotropic thin or slanting shell of constant thickness $2h$ in the three-dimensional Euclidean space R^3. The shell has the middle surface S satisfying necessary conditions of the smoothness with a piecewise-smooth boundary Γ, and the lateral area Σ generated by the motion of the normal $\mathbf{m}$ of the surface S along the contour Γ. I.N. Vekua in [1] obtained a finite system of equilibrium equations for that shell in a vector form within a curvilinear system of coordinates connected with its middle surface. According to the notation of this paper, we represent the finite system of equilibrium equations in the following form

$$\frac{1}{\sqrt{\alpha}}\partial_{\alpha,x}(\sqrt{\alpha}\mathbf{P}^{(k)\alpha}(\mathbf{r}_x)) - \frac{1}{h}\underline{\mathbf{P}^{(k)3}}(\mathbf{r}_x) + \mathbf{\Phi}^{(k)}(\mathbf{r}_x) = 0, \quad \mathbf{r}_x \in S, \quad k = \overline{0,N}, \tag{1}$$

where $\mathbf{r}_x$ is radius-vector of points of middle surface S

$$\mathbf{P}^{(k)i} = \frac{2k+1}{2h}\int_{-h}^{h} P_k\left(\frac{z}{h}\right)\mathbf{P}^i dz,$$

$$\underline{\mathbf{P}^{(k)3}} = (2k+1)\sum_{i=0}^{[(k-1)/2]}\mathbf{P}^{(k-2i-1)3}, \quad k = \overline{1,N}, \quad \underline{\mathbf{P}^{(0)3}} = 0,$$

$$\mathbf{\Phi}^{(k)} = \mathbf{F}^{(k)} + [\mathbf{p}_n^{(+)} + (-1)^k\mathbf{p}_n^{(-)}]\frac{(2k+1)}{2h},$$

$\mathbf{P}^i$ are vector components of the stress tensor, $\mathbf{F}$ is the vector of solid forces, $\mathbf{p}_n^{(\pm)}$ is a given vector of stress on the upper (lower, respectively) face of the shell, $P_k\left(\frac{z}{h}\right)$ is the Legendre polynomial of k-th order, $z \in [-h, h]$ is the transversal coordinate of normal system of coordinates, $\partial_{\alpha,x} \equiv \frac{\partial}{\partial x^\alpha}$ (x^1, x^2 are curvilinear coordinates), a is the determinant of the metric tensor of the middle surface. The Greek subscripts and superscripts in system (1) and further run over values 1, 2, and the Roman ones — 1, 2, 3; repetition of subscripts and superscripts stand for summation (the only exclusion is the order number in the expansion of a vector-function in the Legendre polynomials).

In the context of $(N+1)$-moment approximation the displacement vector for the elements of the shell is determined via the formula (see [1])

$$\mathbf{u}_N = \sum_{k=0}^{N} P_k\left(\frac{z}{h}\right)\mathbf{w}^{(k)}.$$

Then in accordance with the generalized Hooke law

$$\mathbf{P}^{(k)i} = \lambda(\mathbf{r}^m, D_{m,x}\mathbf{w}^{(k)})\mathbf{r}^i + \mu(\mathbf{r}^i, D_{m,x}\mathbf{w}^{(k)})\mathbf{r}^m + \mu a^{im}(D_{m,x}\mathbf{w}^{(k)}, \mathbf{r}^j)\mathbf{r}_j, \tag{2}$$