

The Cauchy Problem for a Generalized Spatial Cauchy–Riemann System

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Abstract—We consider the problem on the analytic continuation of a solution to a generalized Cauchy–Riemann system from its values on a part of the domain boundary.

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INTRODUCTION

We consider the problem on the analytic continuation of a solution to a system of equations (a spatial analog of the generalized Cauchy–Riemann system [1]) to a domain from its values on a part of the boundary of this domain.

The generalized Cauchy–Riemann system is elliptic, and the Cauchy problem for elliptic equations is unstable with regard to small variations of data, i.e., it is ill-posed (the Hadamard example, [2], P. 39). We do not have to prove the existence theorem for ill-posed problems, because the existence is assumed a priori. Moreover, we assume that the solution belongs to some given subset of the functional space. Usually this subset is compact ([3], P. 4). The uniqueness of the solution follows from the general Holmgren theorem ([4], P. 58).

After the unique solvability is established, the theory of ill-posed problems requires to obtain estimates for the conditional stability and to construct regularizing operators. In 1926 T. Carleman ([3], P. 41) obtained a formula that related values of an analytic function of complex variable at points of a domain to its values on a part of the boundary of this domain. On the base of this formula (*ibid.*, P. 34) the notion of the Carleman function for the Cauchy problem for the Laplace equation was introduced. In certain cases it was evaluated. The construction of the Carleman function enables us to construct regularization and to estimate the conditional stability in the mentioned problems.

The interest to this classical ill-posed problem in mathematical physics has not vanished during the recent decades. This direction in studies of properties of solutions to the Cauchy problem for the Laplace equation was begun in the 1950s in works [3], [5–7] and later developed in [8–11].

1. THE STATEMENT OF THE PROBLEM AND THE CARLEMAN MATRICES

Let R^3 be the real three-dimensional Euclidean space,

$$\begin{aligned}x &= (x_1, x_2, x_3), \quad y = (y_1, y_2, y_3) \in R^3, \quad x' = (x_1, x_2, 0), \quad y' = (y_1, y_2, 0), \\s &= (y_1 - x_1)^2 + (y_2 - x_2)^2, \quad r^2 = s + (y_3 - x_3)^2.\end{aligned}$$

Let D be a bounded simply connected domain in R^3 with a piecewise-smooth boundary which consists of a part Σ of the plane $y_3 = 0$ and a smooth surface S lying in the half-space $y_3 > 0$, i.e., $\partial D = \Sigma \cup S$. In the domain D we consider the following system of equations:

$$\operatorname{div} q + (H \cdot q) = 0, \quad \operatorname{rot} q + [q \times H] = 0, \tag{1.1}$$

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